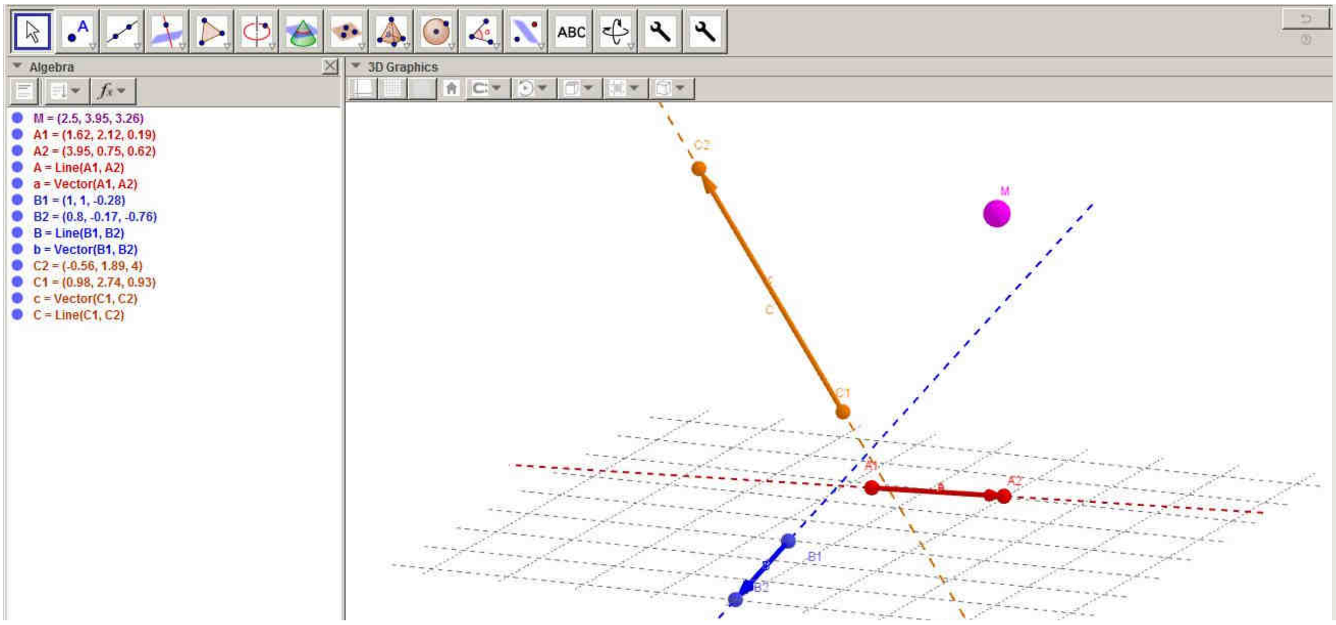
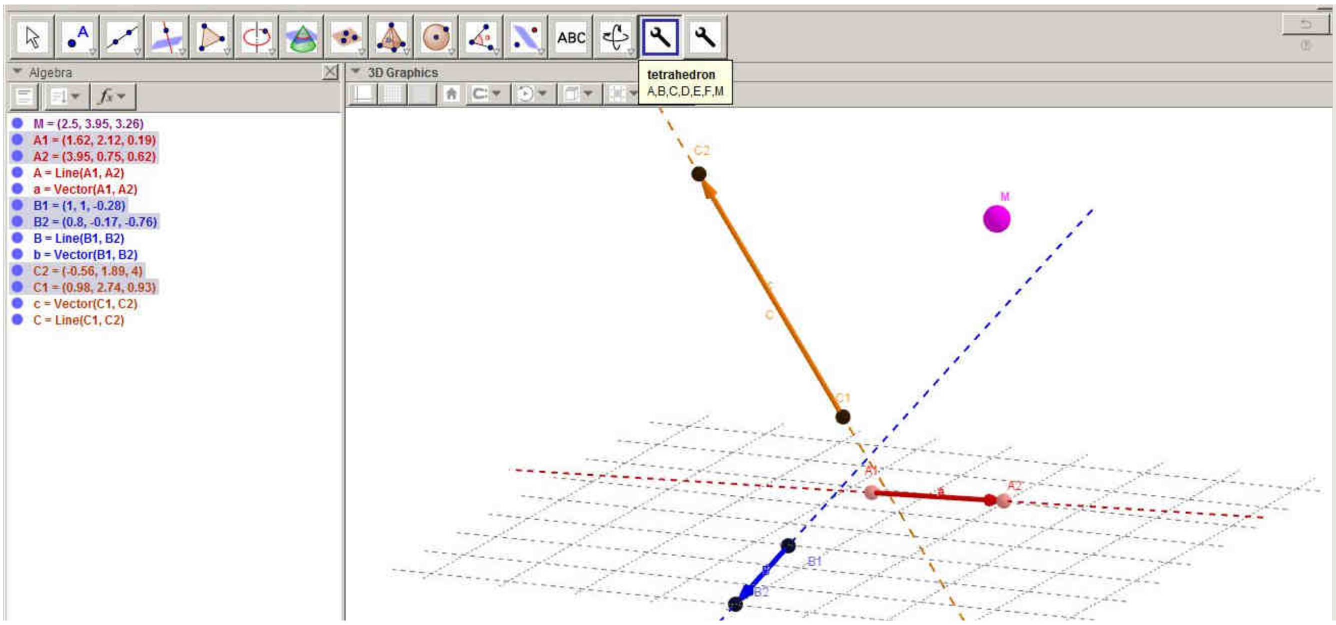
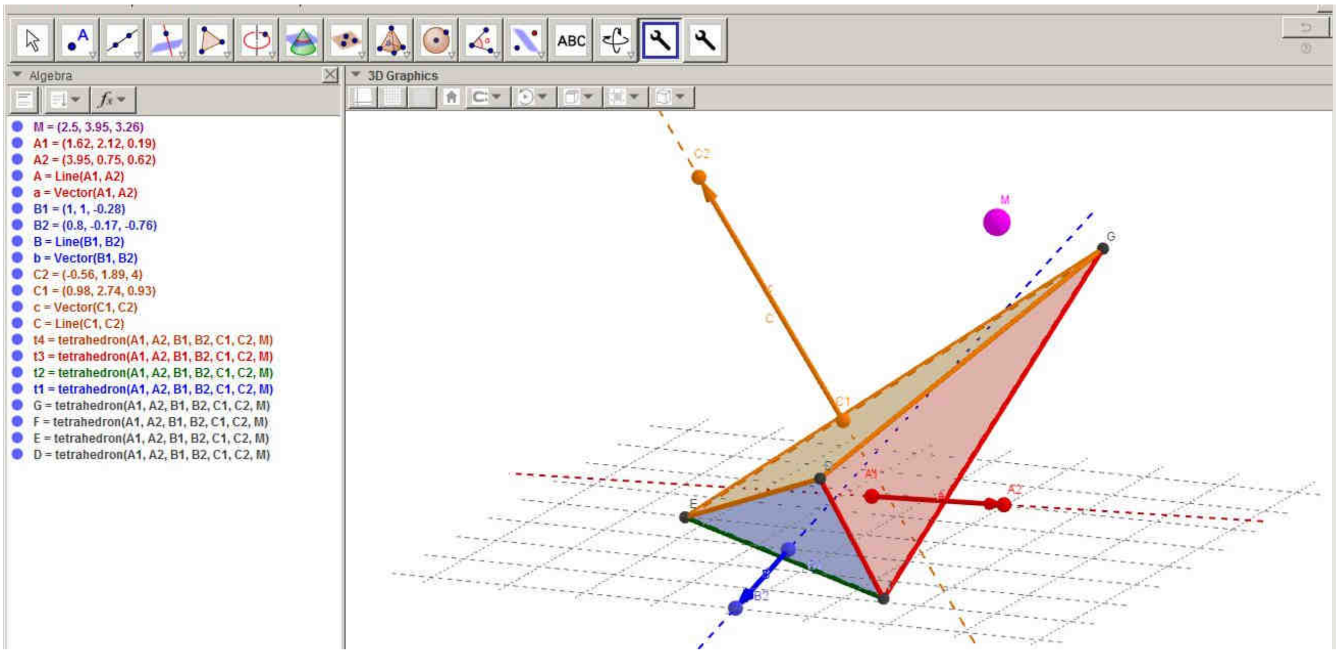
TETRAHEDRON:



1. input: 3 vectors defined by 6 points in space, plus 1 arbitrary point in space



2. select tetrahedron tool
3. select the 6 points defining the 3 vectors

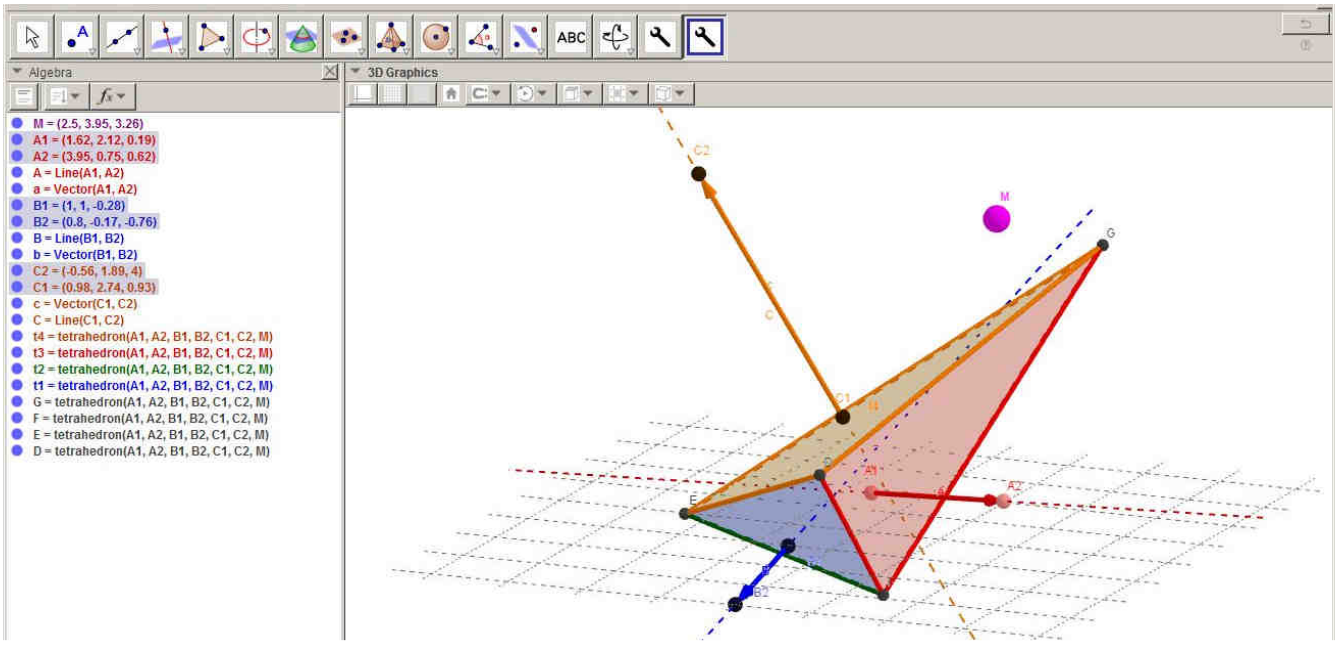


4. select the arbitrary point.

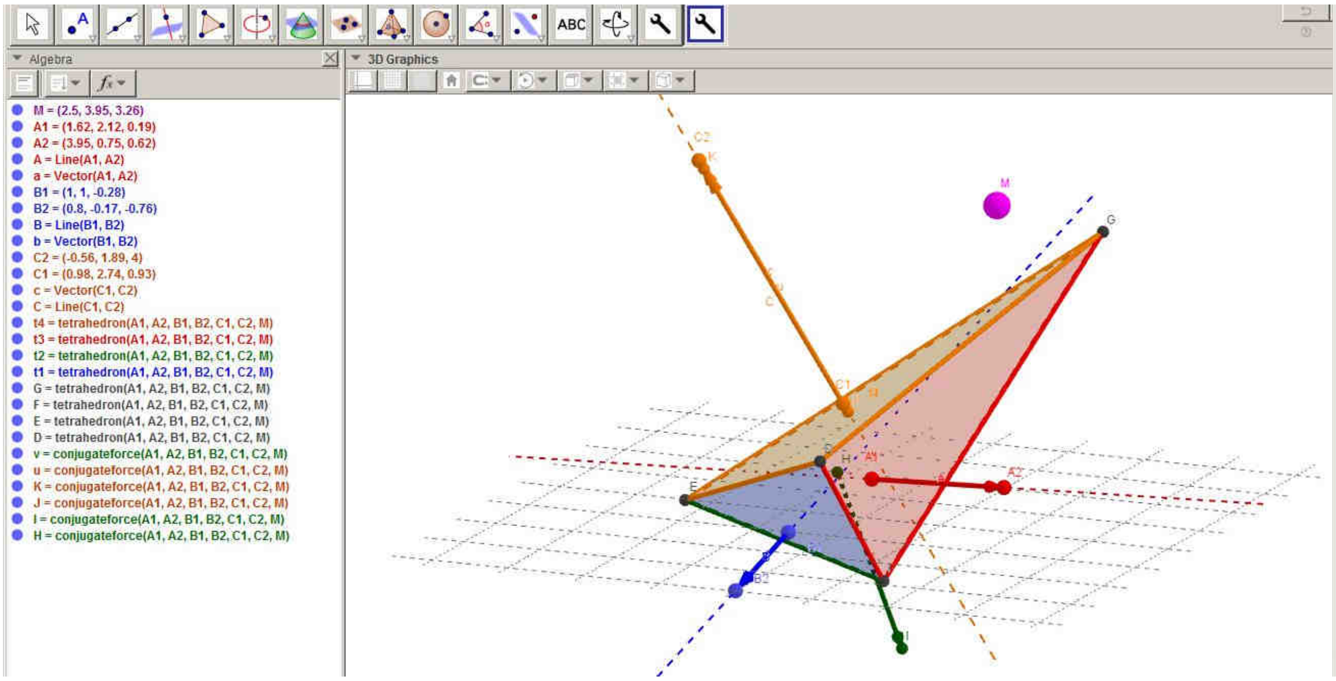
The tetrahedron is such that the oriented area of each face is proportional and perpendicular to each vector. Moving the point M can adjust the tetrahedron.

…Hope someone will come up with a method to build a tetrahedron such that the centroid of each face coincides with the line of action of the corresponding force!

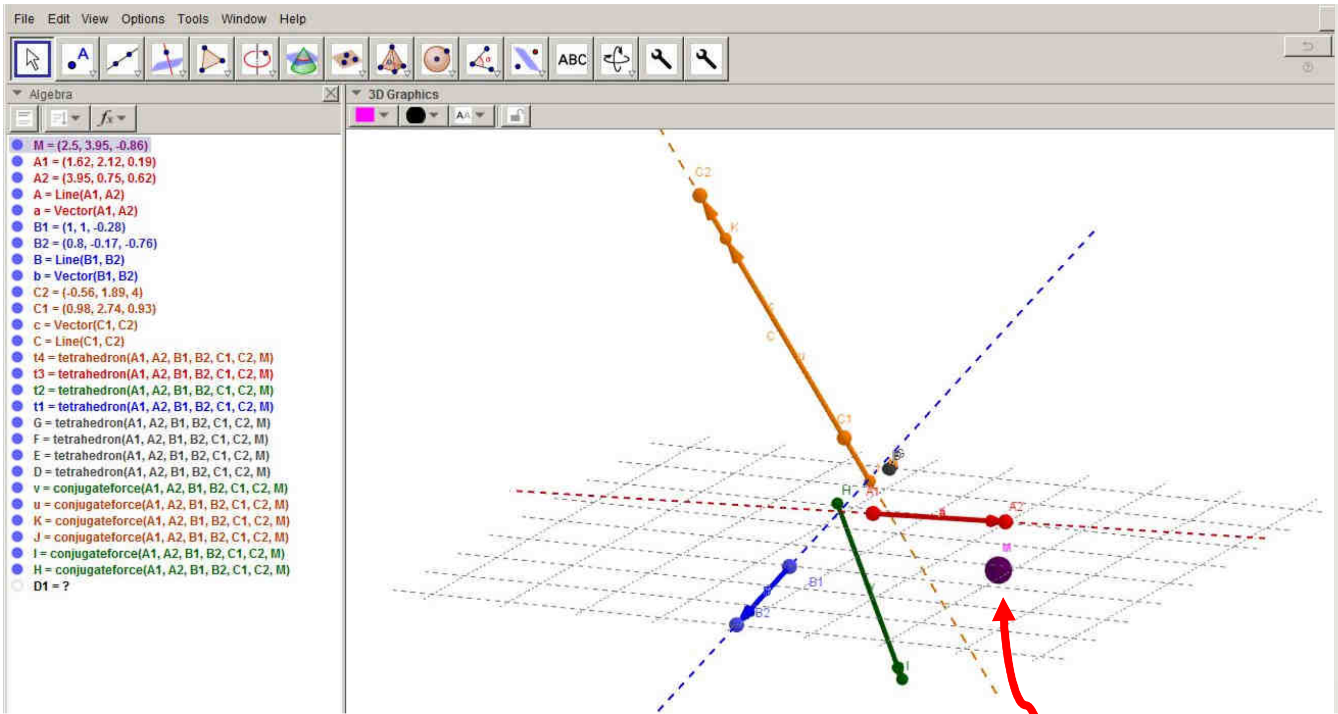
CONJUGATE FORCE:



1. conjugate force of one force: Select the "conjugate force" tool and select the 6 points defining the 3 vectors



2. select the arbitrary point M



3. Moving M you can choose one sort of "scale" of the polyhedron.

One point is particularly interesting, it is when the polyhedron reduces to **one single point**, and beyond, reverses itself. This point may be considered as sort of "Centroid" of the system of 4 forces…

Hope someone will find a method to construct this "central point"!

Note: The 2 tools, conjugate force and tetrahedron are independent. The tetrahedron construction needs the conjugate force but it is included in the tool.

Next pages: Some explanations on the geometrical construction…

① 3 Vectors defined by 6 points:

$\vec{a} = \overrightarrow{A_1A_2}$ (can be coplanar or not)

$\vec{b} = \overrightarrow{B_1B_2}$

$\vec{c} = \overrightarrow{C_1C_2}$

② 3 Lines defined by the same points:

$A = \overline{A_1A_2}$

$B = \overline{B_1B_2}$

$C = \overline{C_1C_2}$

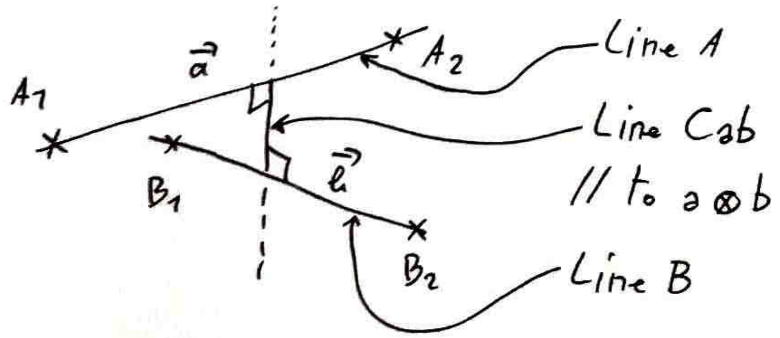
③ Sum & Cross Product of $\vec{a}$ & $\vec{b}$:

$\overrightarrow{S_{ab}} = \vec{a} + \vec{b}$

$\overrightarrow{ab} = \vec{a} \otimes \vec{b}$

④ Locating the Cross Product:

$$(\text{Line})\ C_{ab} = \text{Plane}(A_1, A_2, A_2 + \overrightarrow{ab}) \cap \text{Plane}(B_1, B_2, B_2 + \overrightarrow{ab})$$



⑤ Locating the Resultant:

(Line) R_{ab} = Line through

$$\mathrm{Bary}\left[\left(C_{ab}\cap A,\ \vec{a}\cdot\overrightarrow{S_{ab}}\right),\left(C_{ab}\cap B,\ \vec{b}\cdot\overrightarrow{S_{ab}}\right)\right]$$

// to $\overrightarrow{S_{ab}}$

in GGB:

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Rab=Line[Barycentre({intersect(Cab,A), intersect(Cab,B)},
         {a Sab, b Sab}), Sab]
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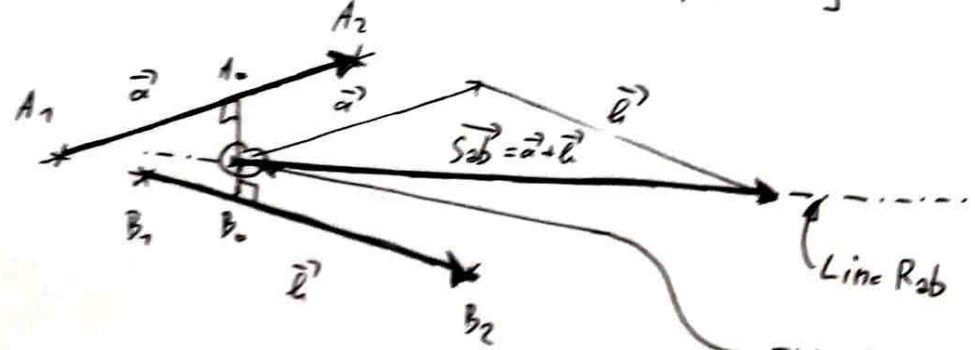


This is the Barycenter of $\left(A_0\ \vec{a}\cdot\overrightarrow{S_{ab}},\ B_0\ \vec{b}\cdot\overrightarrow{S_{ab}}\right)$

⑥ Choosing a location for the conjugate force:

P_{ab} = Plane through M $\perp$ to R_{ab} (M = any point)

$O = R_{ab} \cap P_{ab}$ (Origin selected for the plane) by moving M

⑦ Moving $\vec{c}$ to start from P_{ab}:

$C_1' = C \cap P_{ab}$

$C_2' = C_1' + \vec{c}$

⑧ Finding starting point of $\vec{d}$:

$D_1 = \mathrm{Bary}\left[\left(0, \overrightarrow{S_{ab}} \cdot \overrightarrow{S_{ab}}\right), \left(C_1', \vec{c} \cdot \overrightarrow{S_{ab}}\right)\right]$

⑨ Tracing $\vec{d}$ in position:

$D_2 = D_1 - \vec{a} - \vec{b} - \vec{c}$

$\vec{d} = \overrightarrow{D_1 D_2}$

Any 2 vectors of $(\vec{a}, \vec{b}, \vec{c}, \vec{d})$ have the same screw (resultant + moment) as the other 2.

$(\vec{a}, \vec{b}) = (\vec{c}, \vec{d})$; $(\vec{a}, \vec{c}) = (\vec{b}, \vec{d})$; $(\vec{a}, \vec{d}) = (\vec{b}, \vec{c})$

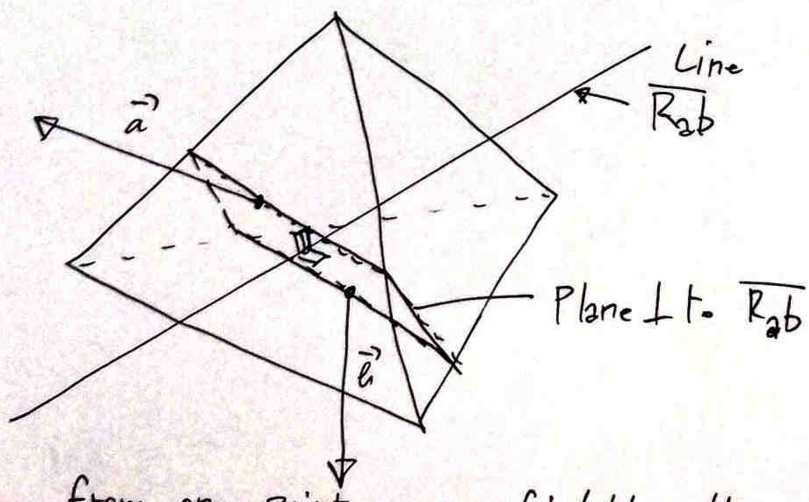
(10) Reconstructing the Tetrahedron :

This is based on the observation that the 3 resultant lines $R_{ab} = R_{cd}$,

$$R_{ac} = R_{bd},$$

$$R_{ad} = R_{bc},$$

define perpendicular planes that intersect the lines of action of 2 forces exactly where the 4 planes of the tetrahedron are :



So from one point one can find the other 3...

References for the theory:

Iva Kodrnja et al., Line geometry and 3D graphic statics, Građevinar 10/2019
DOI: https://doi.org/10.14256/JCE.2725.2019

Jürgen Richter-Gebert, Perspectives on Projective Geometry, Springer 2010

Geogebra Construction of the Conjugate Force and the Tetrahedron:

Laurent Fournier, Kolkata - 28 March 2021