Right, Obtuse, or Acute Triangle?

GeoGebra exploration as to how to tell if a triangle is right, obtuse, or acute from the length of the sides.

- 1. Turn off the axes and grid in GeoGebra.
- 2. Construct a segment and a semicircle with the segment as the diameter. Choose the Semicircle Through 2 Points tool under the Circle icon, then click on the endpoints left to right.



3. Construct a point on the semicircle. Use the Move tool to be sure it is connected to the semicircle. Form a triangle by connecting the point on the semicircle to the end points of the diameter.



4. Use the Measure Angle tool to measure the angle inscribed in the semicircle (in this case angle C) by clicking on point A, then vertex point C, followed by point B.



Move point C around the semicircle. Does the measure of angle C change? Is every angle inscribed in a semicircle a right angle?

5. Construct a line perpendicular to segment AB through point C. Using the Perpendicular Line tool, click on segment AB, followed by point C. Again, drag point C to see that the relationship is maintained.



6. Use the Measure Distance or Length tool to measure the sides of the triangle by clicking on each side of the triangle.



7. First Exploration—What is the relationship between the length of the sides of the triangle in a right triangle? To do this, we will use the algebra portion of GeoGebra. In the Input box of the left column, type in the following. (Use the Keyboard icon at the bottom for the Square key.)

$$AC^2 + BC^2$$

In another input box, type

 AB^2

Notice that the values of each expression. Drag point C around the same circle. As you do this pay attention to the values above in the right-hand column.

Conclusion---What is the relationship of the sides of a right triangle?

8. Construct a point on the perpendicular line outside the circle. Construct segments connecting the point to the endpoints of the diameter.

Use the Measure Distance or Length tool to measure the length of the new segments.

Use the Angle Measure tool to find the measurement of the new angle outside the circle.



9. In the input box, type the following

$${\sf AD}^2 + {\sf BD}^2$$

Followed by

$$AB^2$$

10. Second Exploration—What is the relationship between the lengths of the sides of acute and obtuse triangles? Drag the point above the triangle (in this case, point D) along the perpendicular line. Point C may be moved to any point on the semicircle. As you do this make note of the values for each triangle in the table below.

Acute, obtuse, or right triangle	$AD^2 + BD^2$	AB^2

Conclusion:

Complete the following statements.

A triangle is a right triangle if and only if

A triangle is an acute triangle if and only if

A triangle is an obtuse triangle if and only if