Linear Equations; Function Notation; Graphing Linear Function

8.1 Relations and Functions

1. What letter does the domain of a function represent? These letters are called **inputs**.

2. What letter does the range of a function represent? These letters are called **outputs**.

3. Identify the domain and range of the following relation: (0,1), (2,4), (3,7), (5,4).

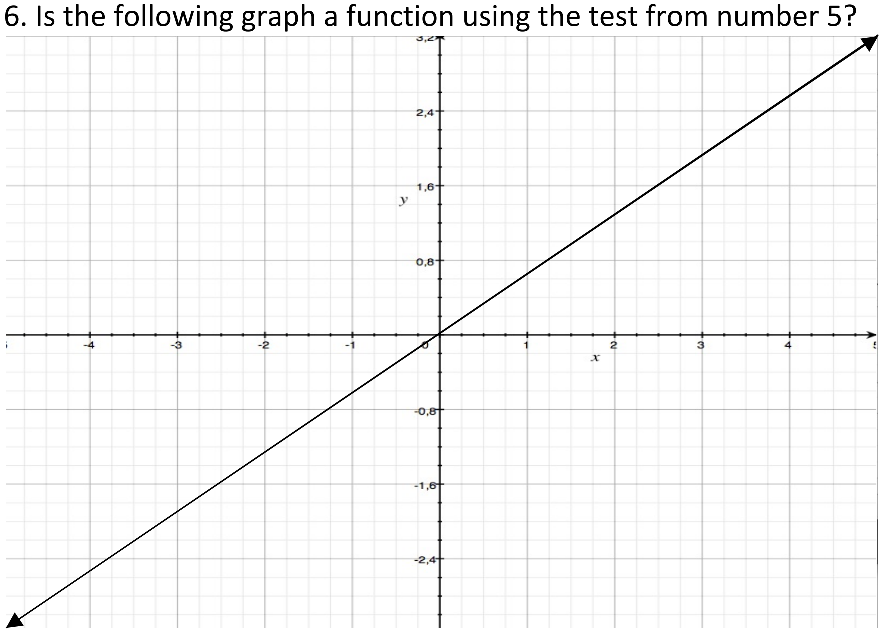
Domain: ___________________ Range: ___________________

4. When there is only one unique output for each input, the relation is called a **function**. Is the following relation a function?

X	**Y**
0	3
1	3
2	4

5. If we have the graph of a function visible to us, we can use the _______________ line test to see if the graph (relation) is a function.

6. Is the following graph a function using the test from number 5? ________



8.2 Linear Functions in Two Variables

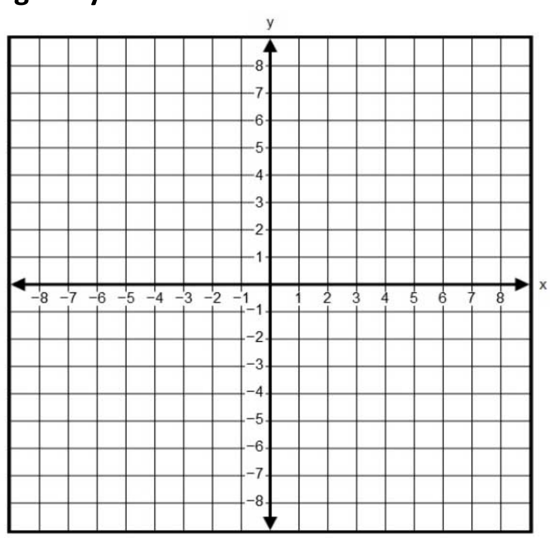
7. Every linear function will be written in two variables, an **x** and a **y**. Here is an example of a linear function in two variables: $4x + y = 15$. A **solution** of an equation is an **ordered pair** of numbers that make a true statement when they are plugged into the function. When **x = 2**, what will **y** be? What is the solution as an ordered pair?

When x = 2, y = _____
The solution as an ordered pair is: _______

8. Tell whether each of the following ordered pairs are solutions to the function: $y = 2x + 1$.

A. (0, 2)   B. (0,1)   C. (2,5)   D. (3,5)

9. Graph the following equation in the 2-dimensional coordinate plane provided: $y = 3x – 2$. Remember, find your 2 ordered pairs first, then plot the points to draw the line! **Choose** your 2 **x** values in order to **get** 2 **y** values.

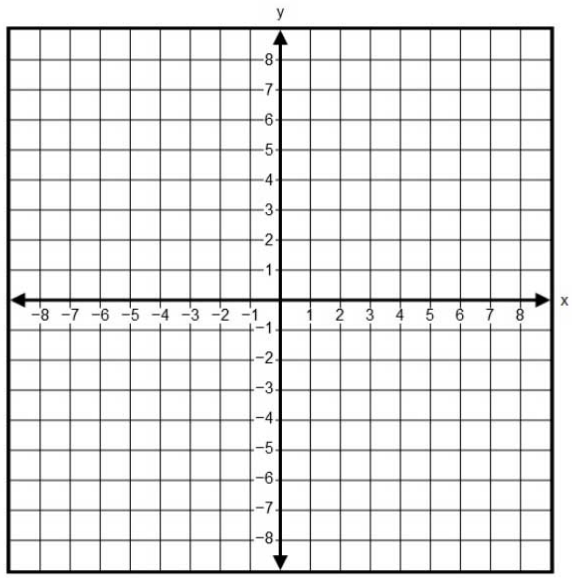


10. An equation that is solved for **y** is in **function form**. Which one equation below is solved for **y** and is therefore in **function form**?

A. $y + 1 = 4x$ B. $y – 4x = -1$ C. $y = 4x – 1$ D. $y – 4x + 1 = 0$

11. Write the following equation in **function form** first and then graph the equation: y + 2 = 2x

Function form: ____________________



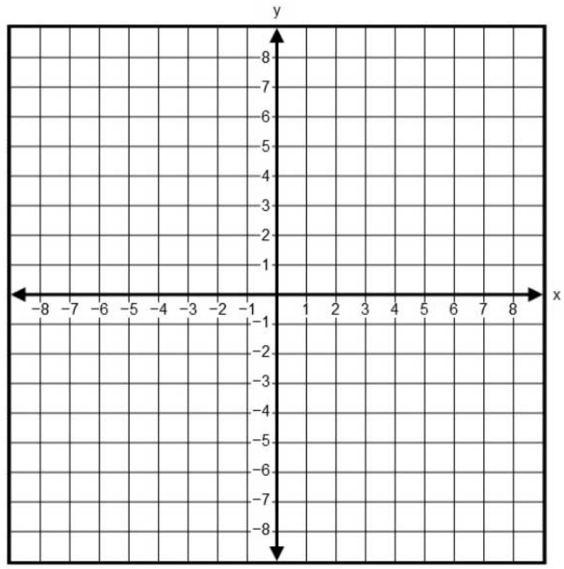
<u>8.3 Using Intercepts</u>

12. An **x-intercept** is a point that lies on (or crosses) the **x-axis**. Likewise, a **y-intercept** is a point that lies on (or crosses) the **y-axis**. Since an **x-intercept** lies on the **x-axis**, its **y-value** will always be 0. Likewise, since a **y-intercept** lies on the **y-axis**, its **x-value** will always be 0.

Find the **x-intercepts** and **y-intercepts** for the following linear functions <u>without graphing them</u>:

A. y = 5x – 10 B. y = -3x + 6 C. y = 2x – 12 D. y = 7x + 42

13. Graph y = 4x + 12 by allowing the 2 points you use to be your **x-intercept** and **y-intercept**:



8.4 The Slope of a Line

14. A line's **slope** is the **ratio** of its vertical change, called the "rise," to its horizontal change, called the "run." When we say that the ratio "compares rise to run," we are saying that "rise" comes first, and "run" comes second. Therefore, "rise" is always our numerator, and "run" is always our denominator in the **slope ratio**.

The slope of a line is usually written with the letter **m**, and it always appears in front of the **x**, such as in the example: y = **2**x + 3. The **2** is the **slope** of this linear equation, and the **3** is the **y-intercept** of the equation, whose ordered pair is (0,3). The linear equation must be written in **function form** in order to spot the **m** correctly. You can change a linear equation into **function form** using the basic rules of algebra.

Find the slope of the following linear equations. They are in function form:

A. y = 3x – 2 B. y = 13x + 26 C. y = -5x – 5 D. y = -9x + 18

15. Find the slope of the following lines. Be sure they are in function form before you do:

A. y = 4x + 8 B. y + 3 = -3x C. y – 7x = 7 D. y – x = 1

16. A line's **slope** is found mechanically by choosing two given points on a line (any 2 points!). Next, we **subtract** those points' **y-values** and write our answer in the numerator of the **slope ratio**. Next, we **subtract** those points' **x-values** and write our answer in the denominator of the **slope ratio**. ***Remember:*** You can't subtract the y-values and the x-values in a different order. You must subtract them both in the same order. If you chose point 2's y-value first, you must choose point 2's x-value first as well. The **slope formula** is $m = \frac{y_2 - y_1}{x_2 - x_1}$.

Find the slope between the following pairs of points:

A. (0,1) and (2,7)　　B. (-3,4) and (3,2)　　C. (-10,10) and (-5,5)

17. Find the slope of the following linear equations. You may either look for **m** or use the slope formula, $m = \frac{y_2 - y_1}{x_2 - x_1}$. If you look for **m**, make sure your linear equation is in **function form**.

A. y = 3x – 1　　B. y – 2 = 3x + 4　　C. y + 4 = 2(x – 2)

18. There are **4 types of slopes**: positive, negative, zero, or undefined. Here is a **chart** for keeping the information separated:

	Positive	**Negative**	**Zero (0)**	**Undefined**
"m"	Positive	Negative	0 in numerator	0 in denominator
Example	Y = 2x + 3	Y = -2x + 3	Y = 3	X = 3
Facts	Slants up to the right	Slants down to the right	Horizontal (flat) line	Vertical (up/down) line

Identify what type of slope is described in each of the following examples:

A. y = -6x + 12　　B. y = 4　　C. y = 6x + 12　　D. x = 4

8.5 Slope-Intercept Form

19. There are two specific forms that we can write a linear equation in, and each form has an advantage. The first of the two, the **slope-intercept form**'s advantage is that it allows us to quickly spot the **slope (m)** and the **y-intercept (b)** for any equation in **function form**.

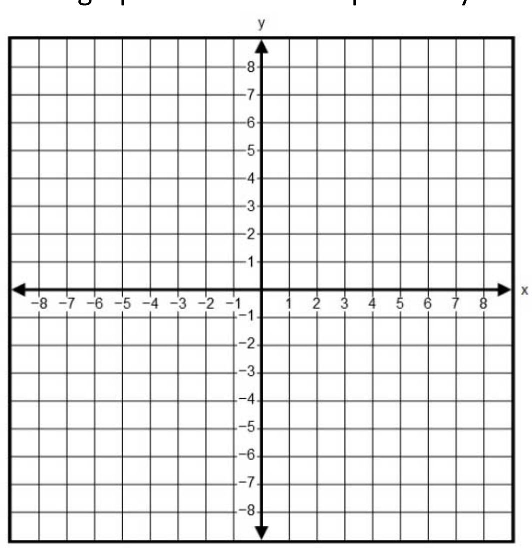
Do not confuse "function form" with "slope-intercept form." Both of these write the linear equation solved for "y," but function form may not have the opposite side (the x terms and the number terms) written as "mx + b." For example:

Function form could be written as: $y = 3x + 4 - 2x + 5$, while slope-intercept form could be written as: $y = x + 9$. The two are very closely related, but **slope-intercept form** can be thought of as the **simplified version** of the **function form**.

Identify both the (1) slope and the (2) y-intercepts of the following linear equations. Write the y-intercepts as ordered pairs with a 0 for the x-value:

A. $y = 8x - 16$ B. $y = x - 3$ C. $y = 5x + 10$ D. $y = 11x$

20. First, identify the slope and the y-intercept of the given linear equation. Then, use that information to sketch a graph of the linear equation: $y = 3x + 6$



21. Sometimes a linear equation can have a **coefficient** in front of the y. When this happens, the **first thing** you want to do is **divide** the **entire equation** (every term) **by** that **coefficient**. In the following linear equations, identify what number you would divide by as your first step.

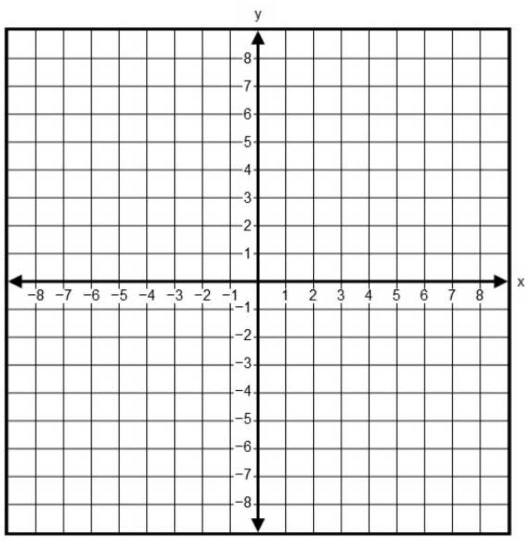
A. $3y = 9x + 12$ B. $-4y + 4x = 4$ C. $-2y = 8x - 2$ D. $9y = 9x + 18$

22. Change the following linear functions into **slope-intercept form**. Remember to watch out for coefficients in front of "y," and make sure your answer matches the form "y = mx + b." When you are done with this, graph problem B.

A. -4x + 8y = 16

B. 30 – 5x = 6y

C. 7y – 21x = 49



<u>8.6 Writing Linear Equations</u>

23. You know enough about how a line is built by this point to write linear equations on your own! Using the **slope-intercept form** to build your equation properly, write the following linear equations, given their conditions, in the space below each prompt:

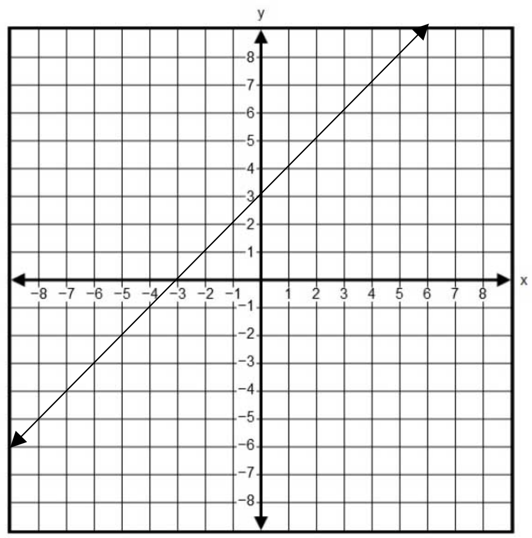
A. Slope of 3 and a y-intercept of 4.

B. Slope of -5 and a y-intercept of 0.

C. Slope of 8 and a y-intercept of -1.

D. Slope of -10 and a y-intercept of -10.

24. Write an equation of the following linear equation based on its graph. (Hint: You can find the slope of a linear function from any two points on the line.)



25. The slopes of some lines are related to the slopes of other lines. When two lines share the same slope but have different y-intercepts, they are said to be parallel lines. Parallel lines' slopes are also parallel to each other. When two lines' slopes cause them to intersect each other at right (90 degree) angles, the slopes of these two lines are said to be perpendicular. Some textbooks use the word "orthogonal" as a synonym to "perpendicular," but they mean the same thing. Perpendicular slopes are related to other by being each other's opposite reciprocals. That means they have an "opposite" sign and are the "flip" version of the original. Whether lines are parallel or perpendicular depends solely upon their **slopes**; <u>their y-intercepts do not play a factor in determining this</u>.

An example of two parallel lines would be: $y = 3x + 9$ and $y = 3x + 4$.

An example of two perpendicular lines would be: $y = -2x + 3$ and $y = \frac{1}{2}x - 10$.

Identify which lines below are parallel and which lines are perpendicular to each other:

A. $y = x$ and $y = x + 2$. ____________________

B. $y = 3x + 3$ and $y = -\frac{1}{3}x - 9$. ____________________

C. $y = \frac{2}{5}x + 4$ and $y = \frac{2}{5}x - 4$. ____________________

D. $y = x$ and $y = -x$. ____________________

8.7 Function Notation

26. **Function notation** is the formal act of replacing "y" in a linear equation with "f(x)." The two mean the exact same thing, and they should always be seen as interchangable synonyms in mathetmatics. Writing "y" as "f(x)" communicates that "y" is some "function of x," or is related in some unique way to x. Consider the following example:

In the equation $y = 6x + 3$, we can replace "y" with "f(x)." Rewriting the equation, we observe $f(x) = 6x + 3$. We are not saying anything any differently; we are just writing the equation in **function notation**. We are saying "If we *do this* to x, then we will *get* f(x) as a result." In the above example, we could say, "If we *multiply x by 6 and add three to that product*, then we will *get* f(x) as a result." <u>It will be a good habit to replace "f(x)" with "y" in your problems until you get completely used to seeing "y" as "f(x)."</u>

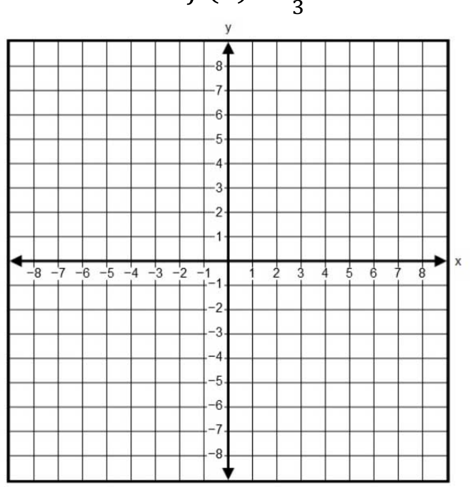
Lastly, the fact that this is called **f**unction notation and an **f**(x) appears in the notation is a coincidence. We can write "y" as "any letter"(x) and still correctly display **function notation**. So, if you see "g(x)" or "h(x)," don't be dismayed! It is the very same thing as saying f(x). We sometimes need different letters because we may deal with multiple equations at once, and we need to keep them separated so that we do our work well.

Answer the following questions implimenting function notation:

A. <u>Find f(x)</u> when x is 5 for the linear equation: $f(x) = 4x + 5$.

B. <u>Find x</u> when f(x) is 15 for the linear equation: $f(x) = x + 8$.

27. Graph the linear function: $f(x) = \frac{2}{3}x + 1$. Remember that f(x) is the same thing as "y."



28. Write a linear function given that f(2) = 8 and f(0) = 6. (Hint: Find two ordered pairs by letting the number in parentheses be your x-value and the answer be your y-value.)

8.9 Graphs of Linear Inequalities

29. The word “inequality” will come up many times in mathematics. The word itself is simply made up of the word “equality,” referring to a function/equation, and the negative prefix “In-,” which negates the first thought. So this is a “non-equality”? Well, we still *see* the line when we graph it, and we still work the problem the same ways. However, we will have a new type of answer and a new type of graph. Our answers will no longer be a single number, but a range of numbers; and our graphs will not just have a line, but a region shaded either above or below that line.

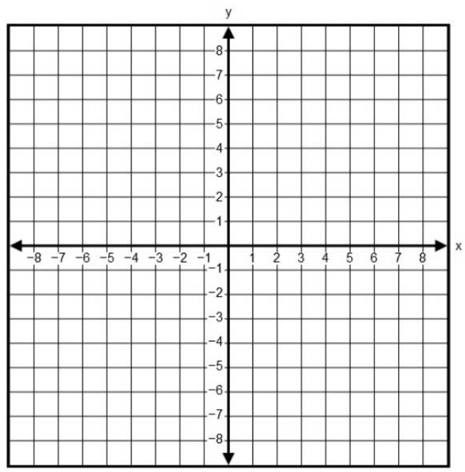
Inequalities answer the same question as equations do, namely: “what values for x and y make this a **true statement**?” However, inequalities answer this question with more than one answer, while equations answer this question with only one unique answer. To solve an inequality, solve it like you would any ordinary equation. The only thing to remember with inequalities is that you will need to change the direction of the inequality symbol if you (1) multiply by a negative number or (2) divide by a negative number.

The symbols are read left to right, so “<” is “less than,” “>” is greater than, “≤” is “less than or equal to,” and “≥” is “greater than or equal to.”

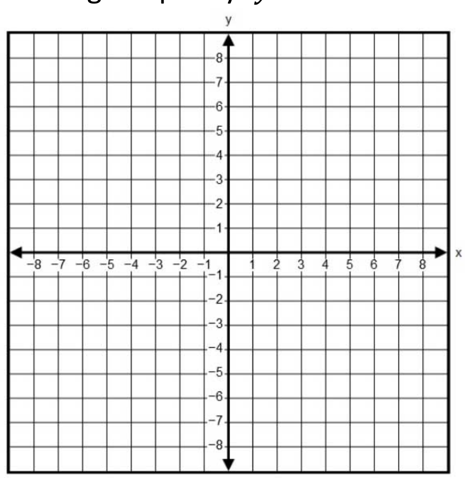
Check to see if the following ordered pairs are solutions to the inequality: $y < 3x + 2$:

A. (0,2)　　B. (3,13)　　C. (4,14)　　D. (-2,-5)　　E. (2,1)

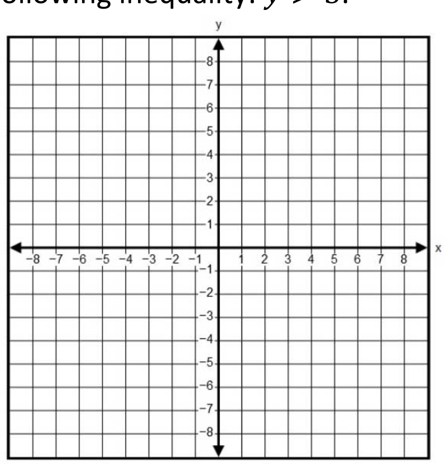
30. Graph the following inequality (Note: If the inequality symbol has an “or equal to” underline, the graph will be solid, and if it does not, it will be dotted. If y > the rest, the shade will be above the line, and if y < the rest, the shade will be below the line.): $y > x + 2$.



31. Graph the following inequality: $y \leq 3x - 1$.



32. Graph the following inequality: $y > 3$.



33. Graph the following inequality: $x \geq 2$.