

Chain rule (AASL/HL)

Title: The Assembly Line - Understanding the Chain Rule in Differential Calculus

Concept: Chain Rule in Calculus



Intuition Pump: Think of the process of manufacturing a product on an assembly line. Each segment of the line modifies the product step-by-step until it reaches its final form. Similarly, the chain rule helps us understand how changing one variable affects another in a composite function, through a series of interlinked functions.

1. Visual Analogy:

- **Assembly Line:** Imagine a car assembly line. The car (function) undergoes various stages of assembly (different functions applied in sequence). For instance, one part of the line installs engines, another paints the car, and so on. Each stage affects the next; the type of engine installed affects how the car is painted (special heat-resistant paint for more powerful engines, perhaps).

- **Linking Functions:** Just as each section of the line impacts the subsequent one, in calculus, if you have a function $g(x)$ inside another function $f(x)$, the chain rule helps you find out how changes in x affect g , and then how these changes affect f . It's like tracing the modifications from one end of the assembly line to the other.

2. Interactive Activity:

- Use a graphical tool or software where students can input composite functions and see how derivatives are calculated step-by-step using the chain rule. For example, they might input $f(g(x)) = \sin(x^2)$ and see how the derivative is calculated.

- Create a relay race game where each student or group represents a different function in a composite. Passing a baton (or another item) represents passing the output of one function to the next, culminating in calculating the derivative at the end of the relay.

3. Real-life Example:

- Discuss how the chain rule is used in physics to calculate the changing rates, like acceleration of a car where speed is a function of time, and time might be a function of some other variable (like the pedal pressure).

4. Mathematical Connection:

- Break down the chain rule formula: If $y=f(u)$ and $u=g(x)$, then the derivative of y with respect to x is $\frac{dy}{dx} = \frac{df}{du} \times \frac{du}{dx}$. This shows how to multiply the rates of change (derivatives)

along the assembly line to get the total effect on the final product.

- Emphasize that each derivative in the chain rule plays a critical role, just like each step on an assembly line is crucial for the final quality of the product.

Using the "Assembly Line" analogy helps students visualize the sequential application of derivatives in the chain rule, making a complex mathematical concept tangible and relatable. This method helps demystify how functions linked together in calculus can be differentiated using the chain rule.