Vektori

i

pravci

obradio Andrija Ninić, 3.d, 2020./2021.

VEKTORI

POČETNA TOČKA (HVATIŠTE)

ZAVRŠNA TOČKA (KRAJ)

VEKTOR = usmjerena dužina

OZNAČAVANJE VEKTORA:

$\overrightarrow{AB}$ ili $\vec{a}$

! VEKTOR SE ODREĐUJE PO:

1. SMJERU

↓

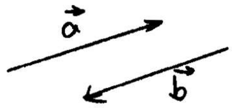
VEKTORI SA ISTIM SMJEROM LEŽE NA PARALELNIM PRAVCIMA (KOLINEARNI)

$\vec{a} \parallel \vec{b}$

2. ORIJENTACIJI

↓

ODREĐUJE SE UKOLIKO SU VEKTORI ISTOGA SMJERA



3. DULJINI

↓

DULJINA VEKTORA JEDNAKA JE DULJINI DUŽINE

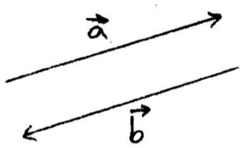
$|\overrightarrow{AB}| = |\overline{AB}|$

$|\overrightarrow{AB}| = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

$\vec{a}^2 = |\vec{a}|^2$

! VEKTORI SU JEDNAKI AKO SE PODUDARAJU U DULJINI, SMJERU I ORIJENTACIJI!

SUPROTNI VEKTORI – VEKTORI KOJI IMAJU ISTU DULJINU I SMJER, A RAZLIČITU ORIJENTACIJU

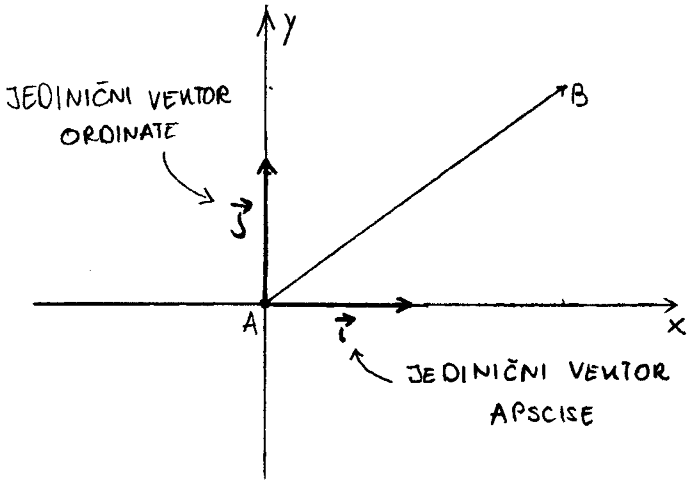


$\vec{a} = -\vec{b}$

NUL-VEKTOR – VEKTOR KOJEMU SE HVATIŠTE I KRAJ PODUDARAJU

- DULJINA MU IZNOSI 0
- OZNAKA: $\vec{0}$

VEKTORI U KARTEZIJEVOM KOORDINATNOM SUSTAVU



$|\vec{j}| = |\vec{i}| = 1$

A(0,0)
B(2, 1.5)

$\overrightarrow{AB} = (2-0)\vec{i} + (1.5-0)\vec{j}$

$\overrightarrow{AB} = 2\vec{i} + 1.5\vec{j}$

ODREĐENJE VEKTORA:

! $\boxed{\overrightarrow{AB} = (x_B - x_A)\vec{i} + (y_B - y_A)\vec{j}}$!

KOORDINATE VEKTORA

!DULJINA VEKTORA PO KOORDINATAMA:

$$\boxed{\begin{array}{l} \vec{a} = x\vec{i} + y\vec{j} \\ |\vec{a}| = \sqrt{x^2 + y^2} \end{array}}$$

$$|\overrightarrow{AB}| = \sqrt{2^2 + 1.5^2} = 2.5$$

! JEDINIČNI VEKTOR VEKTORA:

$$\boxed{\begin{array}{l} \vec{e} = \frac{\vec{a}}{|\vec{a}|} \\ \hline |\vec{e}| = 1 \end{array}}$$

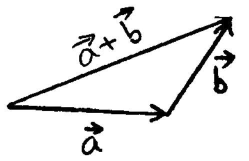
$$\vec{e} = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{2\vec{i} + 1.5\vec{j}}{2.5} = \frac{2}{2.5}\vec{i} + \frac{1.5}{2.5}\vec{j} = 0.8\vec{i} + 0.6\vec{j}$$

ZBRAJANJE VEKTORA

PRAVILO TROKUTA

↓

HVATIŠTE JEDNOG JE KRAJ DRUGOG VEKTORA



$\vec{a} = x_a\vec{i} + y_a\vec{j}$

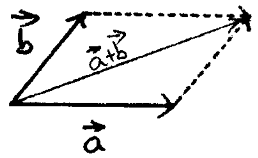
$\vec{b} = x_b\vec{i} + y_b\vec{j}$

$\vec{a}+\vec{b} = (x_a + x_b)\vec{i} + (y_a + y_b)\vec{j}$

PRAVILO PARALELOGRAMA

↓

HVATIŠTA VEKTORA SU ISTE TOČKE



MNOŽENJE VEKTORA SKALAROM

$\vec{a} = x_a \vec{i} + y_a \vec{j}$

$\lambda \cdot \vec{a} = \lambda x_a \vec{i} + \lambda y_a \vec{j}$

$\lambda < 0 \rightarrow$ VEKTOR JE SUPROTNE ORIJENTACIJE

$\lambda \in \langle -\infty, -1 \rangle \cup \langle 1, +\infty \rangle$

VEKTOR JE DULJI

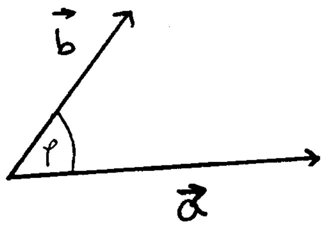
$\lambda \in \langle -1, 1 \rangle$

VEKTOR JE KRAĆI

$\lambda \in \{-1, 1\}$

VEKTOR JE JEDNAKE DULJINE

SKALARNI UMNOŽAK



$$\boxed{\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\varphi}$$

$\alpha < 90^\circ$ $\quad \vec{a} \cdot \vec{b} > 0$

$\alpha = 90^\circ$ $\quad \vec{a} \cdot \vec{b} = 0$

$\alpha > 0$ $\quad \vec{a} \cdot \vec{b} < 0$

$$\left.\begin{array}{l} \vec{a} = x_a \cdot \vec{i} + y_a \cdot \vec{j} \\ \vec{b} = x_b \cdot \vec{i} + y_b \cdot \vec{j} \end{array}\right\} \quad \boxed{\vec{a} \cdot \vec{b} = x_a \cdot x_b + y_a \cdot y_b}$$

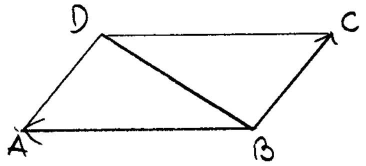
UVJET KOLINEARNOSTI

VEKTORI SU KOLINEARNI AKO VRIJEDI:

$\vec{a} = k \cdot \vec{b}$

$|\vec{a}| = k \cdot |\vec{b}|$

Točke $A(-1,-1)$, $B(3,-2)$ i $C(5,2)$ tri su uzastopna vrha paralelograma $ABCD$. Kolika je duljina dijagonale $\overline{BD}$?



$A(-1,-1)$
$B(3,-2)$
$C(5,2)$

$|\overline{BD}| = ?$

$\vec{BD} = \vec{BA} + \vec{BC}$

$\vec{BA} = (-1-3)\vec{i} + (-1-(-2))\vec{j} = -4\vec{i} + \vec{j}$

$\vec{BC} = (5-3)\vec{i} + (2-(-2))\vec{j} = 2\vec{i} + 4\vec{j}$

$\vec{BD} = -4\vec{i} + \vec{j} + 2\vec{i} + 4\vec{j}$

$\vec{BD} = -2\vec{i} + 5\vec{j}$

$|\vec{BD}| = |\overline{BD}| = \sqrt{(-2)^2 + 5^2}$

$\boxed{|\overline{BD}| = \sqrt{29} \approx 5.39}$

Odredi vektor $\vec{v}$ kolinearan s vektorom $\overrightarrow{AB}$, gdje je $A(2,-1)$, $B(-1,3)$ ako je $|\vec{v}| = 20$.

$\left.\begin{matrix} A(2,-1) \\ B(-1,3) \end{matrix}\right\} \vec{AB} = (-1-2)\vec{i} + (3-(-1))\vec{j} = -3\vec{i} + 4\vec{j}$

$|\vec{v}| = 20$

$|\vec{AB}| = \sqrt{(-3)^2 + 4^2}$

$\vec{v} = k \cdot \vec{AB}$ ili $\vec{AB} = k \cdot \vec{v}$

$|\vec{AB}| = 5$

$\vec{v} = ?$

$|\vec{v}| = k \cdot |\vec{AB}|$

$20 = k \cdot 5$

$k = 4$

$\vec{v} = 4(-3\vec{i} + 4\vec{j})$

$\boxed{\vec{v}_1 = -12\vec{i} + 16\vec{j}}$

$|\vec{AB}| = k \cdot |\vec{v}|$

$5 = k \cdot 20$

$k = \frac{1}{4}$

$\vec{v} = \frac{1}{4}(-3\vec{i} + 4\vec{j})$

$\boxed{\vec{v}_2 = -\frac{3}{4}\vec{i} + \vec{j}}$

Odredi nepoznatu koordinatu točke $B(x, 2)$ tako da duljina vektora $\overrightarrow{AB}$, $A(-3,1)$ bude jednaka $5\sqrt{2}$.

$$\left.\begin{array}{l} B(x,2) \\ A(-3,1) \end{array}\right\} \overrightarrow{AB} = [x-(-3)]\vec{i} + (2-1)\vec{j} = (x+3)\vec{i} + \vec{j}$$

$|\overrightarrow{AB}| = 5\sqrt{2}$

$x = ?$

$$|\overrightarrow{AB}| = \sqrt{(x+3)^2 + 1^2}$$

$$5\sqrt{2} = \sqrt{x^2+6x+10} \quad /^2$$

$$50 = x^2+6x+10$$

$$x^2+6x-40=0$$

$$\boxed{x_1 = 4} \qquad \boxed{x_2 = -10}$$

P: $5\sqrt{2} = \sqrt{(4+3)^2+1}$

$5\sqrt{2} = \sqrt{50}$

$5\sqrt{2} = 5\sqrt{2}$ ✓

P: $5\sqrt{2} = \sqrt{(-10+3)^2+1}$

$5\sqrt{2} = \sqrt{50}$

$5\sqrt{2} = 5\sqrt{2}$ ✓

Ako je $\vec{a} = 5\vec{i} - 12\vec{j}$, $\vec{b} = 4\vec{i} + 9\vec{j}$, koliki kut zatvaraju vektori $\vec{a} + \vec{b}$ i $\vec{a}$?

$\vec{a} = 5\vec{i} - 12\vec{j} \rightarrow |\vec{a}| = \sqrt{5^2 + (-12)^2} = 13$

$\vec{b} = 4\vec{i} + 9\vec{j}$

$\measuredangle(\vec{a}+\vec{b}, \vec{a}) = ?$

$$\vec{a}+\vec{b} = 5\vec{i} - 12\vec{j} + 4\vec{i} + 9\vec{j} = 9\vec{i} - 3\vec{j} \rightarrow |\vec{a}+\vec{b}| = \sqrt{9^2 + (-3)^2} = 3\sqrt{10}$$

$$(\vec{a}+\vec{b}) \cdot \vec{a} = |\vec{a}+\vec{b}| \cdot |\vec{a}| \cdot \cos \measuredangle(\vec{a}+\vec{b}, \vec{a})$$

$$\cos\measuredangle = \frac{(\vec{a}+\vec{b}) \cdot \vec{a}}{|\vec{a}+\vec{b}| \cdot |\vec{a}|} = \frac{9 \cdot 5 + (-3) \cdot (-12)}{3\sqrt{10} \cdot 13}$$

$$\cos\measuredangle = \frac{81}{39\sqrt{10}} = \frac{27\sqrt{10}}{130}$$

$$\boxed{\measuredangle(\vec{a}+\vec{b}, \vec{a}) = 48^\circ\ 56'\ 43''}$$

Kolika je duljina vektora $\vec{v} = 3\vec{a} + 2\vec{b}$ ako je $|\vec{a}| = 2$, $|\vec{b}| = \sqrt{2}$ te kut između vektora $\vec{a}$ i $\vec{b}$ ima mjeru $\frac{3\pi}{4}$.

$\vec{v} = 3 \cdot \vec{a} + 2 \cdot \vec{b}$

$|\vec{a}| = 2$

$|\vec{b}| = \sqrt{2}$

$\measuredangle(\vec{a}, \vec{b}) = \frac{3\pi}{4}$

$|\vec{v}| = ?$

$\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\measuredangle(\vec{a}, \vec{b})$

$\vec{a} \cdot \vec{b} = 2\sqrt{2} \cdot \cos\left(\frac{3\pi}{4}\right)$

$\vec{a} \cdot \vec{b} = \cancel{2}\sqrt{2} \cdot \left(-\frac{\sqrt{2}}{\cancel{2}}\right)$

$\vec{a} \cdot \vec{b} = -2$

$\vec{v} = 3\vec{a} + 2\vec{b} \quad /^2$

$\vec{v}^2 = 9\vec{a}^2 + 12\vec{a} \cdot \vec{b} + 4\vec{b}^2$

$\vec{v}^2 = 9 \cdot 2^2 + 12 \cdot (-2) + 4 \cdot (\sqrt{2})^2$

$\vec{v}^2 = 20 \quad /\sqrt{\ }$

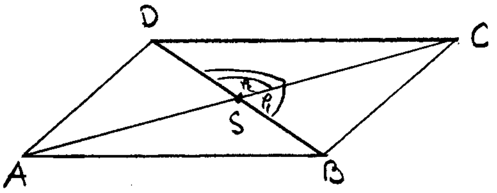
$\boxed{|\vec{v}| = 2\sqrt{5}}$

Odredi kut između dijagonala paralelograma $ABCD$ ako je $\overrightarrow{AB} = 4\vec{i} - 3\vec{j}$, $\overrightarrow{AD} = 6\vec{i} + \vec{j}$.

$\overrightarrow{AB} = 4\vec{i} - 3\vec{j}$

$\overrightarrow{AD} = 6\vec{i} + \vec{j}$

$\varphi = ?$



$\overrightarrow{AC} = \overrightarrow{AB} + \overrightarrow{AD}$

$\overrightarrow{AC} = 4\vec{i} - 3\vec{j} + 6\vec{i} + \vec{j}$

$\overrightarrow{AC} = 10\vec{i} - 2\vec{j}$

$\overrightarrow{SC} = \frac{1}{2}\overrightarrow{AC}$

$\overrightarrow{SC} = 5\vec{i} - \vec{j} \rightarrow |\overrightarrow{SC}| = \sqrt{5^2 + (-1)^2} = \sqrt{26}$

$\overrightarrow{DB} = \overrightarrow{DA} + \overrightarrow{AB}$

$\overrightarrow{DB} = -\overrightarrow{AD} + \overrightarrow{AB}$

$\overrightarrow{DB} = -6\vec{i} - \vec{j} + 4\vec{i} - 3\vec{j}$

$\overrightarrow{DB} = -2\vec{i} - 4\vec{j}$

$\overrightarrow{SB} = \frac{1}{2}\overrightarrow{DB}$

$\overrightarrow{SB} = -\vec{i} - 2\vec{j} \Rightarrow |\overrightarrow{SB}| = \sqrt{(-1)^2 + (-2)^2} = \sqrt{5}$

$$\overrightarrow{SC} \cdot \overrightarrow{SB} = |\overrightarrow{SC}| \cdot |\overrightarrow{SB}| \cdot \cos\varphi$$

$$\cos\varphi = \frac{\overrightarrow{SC} \cdot \overrightarrow{SB}}{|\overrightarrow{SC}| \cdot |\overrightarrow{SB}|}$$

$$\cos\varphi = \frac{5 \cdot (-1) + (-1)(-2)}{\sqrt{26} \cdot \sqrt{5}}$$

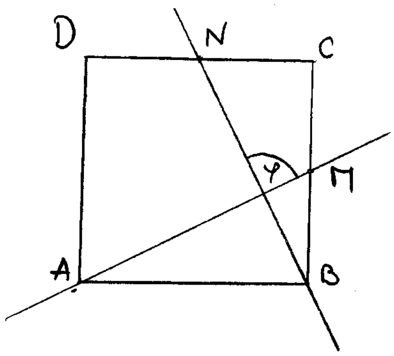
$$\cos\varphi = \frac{-3}{\sqrt{130}} = -\frac{3\sqrt{130}}{130}$$

$\varphi_1 = 105^\circ\ 15'\ 18''$

$\varphi_2 = 180^\circ - \varphi_1$

$\varphi_2 = 74^\circ\ 44'\ 42''$

Točka M polovište je stranice $\overline{BC}$, a točka N stranice $\overline{CD}$ kvadrata $ABCD$. Dokaži da su pravci AM i BN okomiti.



$$\varphi \stackrel{?}{=} 90°$$

$$|AB| = |BC| = |CD| = |AD|$$

$$\vec{AM} = \vec{AB} + \vec{BM}$$

$$\vec{AM} = \vec{AB} + \frac{1}{2}\vec{BC}$$

$$\vec{BN} = \vec{BC} + \vec{CN}$$

$$\vec{BN} = \vec{BC} + \frac{1}{2}\vec{CD}$$

$$\vec{BN} = \vec{BC} - \frac{1}{2}\vec{AB}$$

$$\vec{AM} \cdot \vec{BN} = \vec{AB} \cdot \left(-\frac{1}{2}\vec{AB}\right) + \frac{1}{2}\vec{BC} \cdot \vec{BC}$$

$$= -\frac{1}{2}\vec{AB}^2 + \frac{1}{2}\vec{BC}^2$$

$$= -\frac{1}{2}|\vec{AB}|^2 + \frac{1}{2}|\vec{BC}|^2$$

$$|\vec{AB}| = |\vec{BC}|$$

$$\vec{AM} \cdot \vec{BN} = 0$$

$$\cos\varphi = 0$$

$$\varphi = 90°, \quad \varphi = \frac{\pi}{2}$$

Odredi jedinični vektor okomit na vektor $\overrightarrow{AB}$ ako je $A(-2,3)$ i $B(-4,2)$.

$$\left.\begin{array}{l} A(-2,3) \\ B(-4,2) \end{array}\right\} \overrightarrow{AB} = -2\vec{i} - \vec{j} \qquad \vec{e} = x_e \cdot \vec{i} + y_e \cdot \vec{j}$$

$$\overrightarrow{AB} \perp \vec{e} \rightarrow \overrightarrow{AB} \cdot \vec{e} = 0 \rightarrow -2 \cdot x_e + (-1) \cdot y_e = 0$$

$$\vec{e} = ?$$

$$-2x_e - y_e = 0$$

$$y_e = -2x_e$$

$$|\vec{e}| = 1$$

$$\sqrt{x_e^2 + y_e^2} = 1$$

$$\sqrt{x_e^2 + (-2x_e)^2} = 1$$

$$\sqrt{5x_e^2} = 1 \quad /^2$$

$$5x_e^2 = 1$$

$$x_e^2 = \frac{1}{5}$$

$$x_{e_1} = \frac{\sqrt{5}}{5} \qquad x_{e_2} = -\frac{\sqrt{5}}{5}$$

$$y_e = -2x_e \qquad y_e = -2x_e$$

$$y_{e_1} = -\frac{2\sqrt{5}}{5} \qquad y_{e_2} = \frac{2\sqrt{5}}{5}$$

$$\vec{e}_1 = \frac{\sqrt{5}}{5}\vec{i} - \frac{2\sqrt{5}}{5}\vec{j} \qquad \vec{e}_2 = -\frac{\sqrt{5}}{5}\vec{i} + \frac{2\sqrt{5}}{5}\vec{j}$$

PRAVAC

! IMPLICITNI OBLIK JEDNADŽBE PRAVCA

$$Ax + By + C = 0$$

$$A, B, C \in \mathbb{R}$$

$$A \text{ ili } B \neq 0$$

! EKSPLICITNI OBLIK JEDNADŽBE PRAVCA

$$y = k \cdot x + l$$

KOEFICIJENT SMJERA

ODSJEČAK NA OS ORDINATU

$$k = \frac{y_2 - y_1}{x_2 - x_1}$$

PRIKLONI KUT: $\operatorname{tg} \varphi = k$

→ KUT KOJI PRAVAC ZATVARA SA X-OSI

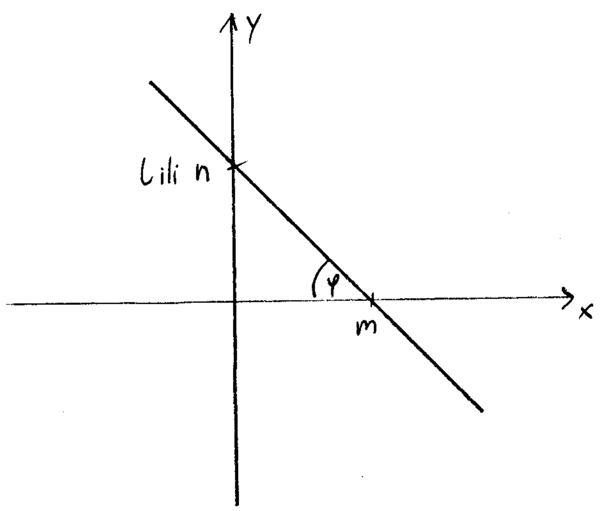
! SEGMENTNI OBLIK JEDNADŽBE PRAVCA

$$\frac{x}{m} + \frac{y}{n} = 1$$

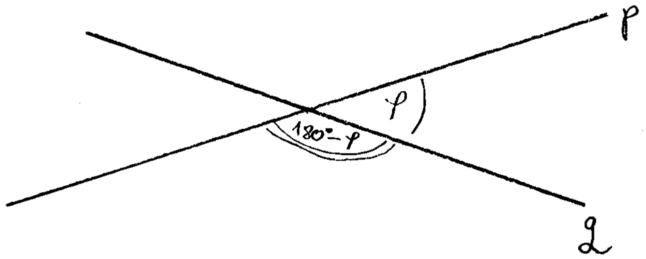
SJECIŠTE SA X-OSI

SJECIŠTE SA Y-OSI

SKICA:



KUT DVAJU PRAVCA



p... $y = k_1 \cdot x + l_1$

q... $y = k_2 \cdot x + l_2$

$$\boxed{\operatorname{tg} \varphi = \left| \frac{k_2 - k_1}{1 + k_1 \cdot k_2} \right|}$$

PARALELNOST I OKOMITOST PRAVACA

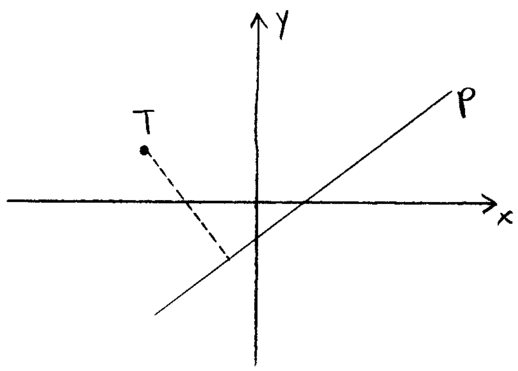
PARALELNOST

$$\boxed{k_1 = k_2}$$

OKOMITOST

$$\boxed{k_1 = -\frac{1}{k_2}}$$

UDALJENOST TOČKE OD PRAVCA

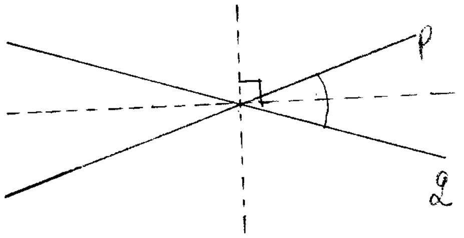


$T(x_T, y_T)$

p... $Ax + By + C = 0$

$$\boxed{d(T, p) = \frac{|A \cdot x_T + B \cdot y_T + C|}{\sqrt{A^2 + B^2}}}$$

! SIMETRALA KUTA IZMEĐU PRAVACA



p... $A_1 x + B_1 y + C_1 = 0$

q... $A_2 x + B_2 y + C_2 = 0$

$$\frac{|A_1 x + B_1 y + C_1|}{\sqrt{A_1^2 + B_1^2}} = \frac{|A_2 x + B_2 y + C_2|}{\sqrt{A_2^2 + B_2^2}}$$

Odredi oblik jednadžbe $2x - 3y + 6 = 0$ i pretvori ju u druga dva.

$2x - 3y + 6 = 0$

IMPLICITNI

EKSPLICITNI: $2x - 3y + 6 = 0$

$-3y = -2x - 6 \quad /\cdot\left(-\frac{1}{3}\right)$

$y = \frac{2}{3} + 2$

SEGMENTNI: $2x - 3y + 6 = 0$

$2x - 3y = -6 \quad /\cdot\left(-\frac{1}{6}\right)$

$-\frac{1}{3}x + \frac{1}{2}y = 1$

$\frac{x}{-3} + \frac{y}{2} = 1$

11) Pravci $2x + 3my - 8 = 0$, $mx + y + 3 = 0$ i $3x - y - 5 = 0$ prolaze jednom točkom u ravnini. Koja je to točka?

$2x + 3my - 8 = 0$

$mx + y + 3 = 0$

$3x - y - 5 = 0$

S ?

$\begin{cases} 2x + 3my - 8 = 0 \\ mx + y + 3 = 0 \end{cases} \Rightarrow x = -\frac{y+3}{m}$

$2\cdot\left(-\frac{y+3}{m}\right) + 3my - 8 = 0$

$\frac{-2y-6}{m} + 3my - 8 = 0 \quad /\cdot m$

$-2y - 6 + 3y\,m^2 - 8m = 0$

$-2y + 3ym^2 = 6 + 8m$

$y(3m^2 - 2) = 6 + 8m$

$y = \frac{6+8m}{3m^2-2}$

$x = -\frac{\frac{6+8m}{3m^2-2} + 3}{\frac{m}{1}}$

$x = -\frac{\frac{6+8m+3(3m^2-2)}{3m^2-2}}{\frac{m}{1}}$

$x = -\frac{\cancel{6}+8m+9m^2\cancel{-6}}{m(3m^2-2)}$

$x = -\frac{\cancel{m}(8+9m)}{\cancel{m}(3m^2-2)}$

$x = \frac{-9m-8}{3m^2-2}$

$3\cdot\left(\frac{-9m-8}{3m^2-2}\right) - \frac{6+8m}{3m^2-2} - 5 = 0$

$\frac{-27m-24}{3m^2-2} - \frac{6+8m}{3m^2-2} - 5 = 0$

$\frac{-27m-24-6-8m-5(3m^2-2)}{3m^2-2} = 0$

$\frac{-15m^2-35m-20}{3m^2-2} = 0 \quad /\cdot(3m^2-2)\neq 0$

$m \neq \pm\frac{\sqrt{6}}{3}$

$-15m^2 - 35m - 20 = 0 \quad /\cdot\left(-\frac{1}{5}\right)$

$3m^2 + 7m + 4 = 0$

$m_1 = -1$

$\begin{cases} -x + y + 3 = 0 \\ 3x - y - 5 = 0 \end{cases} \quad /+$

$2x - 2 = 0$

$x = 1$

$-1 + y + 3 = 0$

$y = -2$

$T_1(1, -2)$

$m_2 = -\frac{4}{3}$

$\begin{cases} -\frac{4}{3}x + y + 3 = 0 \\ 3x - y - 5 = 0 \end{cases} \quad /+$

$\frac{5}{3}x - 2 = 0$

$x = \frac{6}{5}$

$-\frac{4}{3}\cdot\frac{6}{5} + y + 3 = 0$

$y = -\frac{7}{5}$

$T_2\left(\frac{6}{5}, -\frac{7}{5}\right)$

Točke $A(-6,2)$ i $B(2,-2)$ dva su vrha trokuta ABC, a točka $H(1,2)$ njegov je ortocentar. Odredi koordinate vrha C ovog trokuta.

$$\left.\begin{array}{l} A(-6,2) \\ B(2,-2) \end{array}\right\} k_{AB} = \frac{-2-2}{2-(-6)} = -\frac{1}{2} \Rightarrow k_{CH} = 2$$

$H(1,2)$

$C = ?$

$$k_{BH} = \frac{2-(-2)}{1-2} = -4$$

$$k_{AC} = \frac{1}{4}$$

$p_{CH} \ldots \ y = 2x + l$

$2 = 2 \cdot 1 + l$

$l = 0$

$p_{CH} \ldots \ y = 2x$

$p_{AC} \ldots \ y = \frac{1}{4}x + l$

$2 = \frac{1}{4} \cdot (-6) + l$

$l = \frac{7}{2}$

$p_{AC} \ldots \ y = \frac{1}{4}x + \frac{7}{2}$

$$\begin{cases} y = 2x \\ y = \frac{1}{4}x + \frac{7}{2} \end{cases}$$

$2x = \frac{1}{4}x + \frac{7}{2}$

$\frac{7}{4}x = \frac{7}{2} \ / \cdot \frac{4}{7}$

$x = 2$

$y = 2x$

$y = 4$

$\boxed{C(2,4)}$

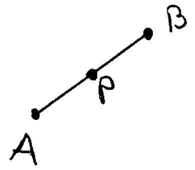
Odredi jednadžbu simetrale dužine $\overline{AB}$ ako je $A(-3,0)$ i $B(5,2)$.

$$\left.\begin{array}{l} A(-3,0) \\ B(5,2) \end{array}\right\} k = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2-0}{5-(-3)} = \frac{1}{4}$$

$p \perp \overline{AB}$

$p \ldots ?$

$k_p = -4$



$x_P = \frac{x_A + x_B}{2}$ $\quad y_P = \frac{y_A + y_B}{2}$

$x_P = 1$ $\quad y_P = 1$

$P(1,1)$

$p \ldots \ y = kx + l$

$y = -4x + l$

$1 = -4 \cdot 1 + l$

$l = 5$

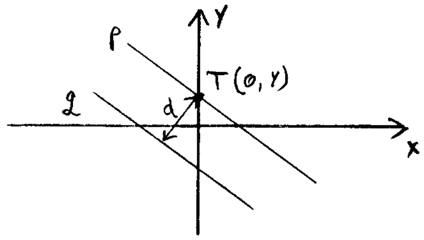
$\boxed{y = -4x + 5}$

Kolika je površina kvadrata kojem dvije stranice pripadaju pravcima $3x + 2y - 5 = 0$ i $3x + 2y + 8 = 0$?

$p \ldots \; 3x+2y-5=0$

$q \ldots \; 3x+2y+8=0$

$P_{\square} = ?$



$T(0, y)$

$3 \cdot 0 + 2y - 5 = 0$

$2y = 5$

$y = \frac{5}{2}$

$$d(T, q) = \frac{|3 \cdot 0 + 2 \cdot \frac{5}{2} + 8|}{\sqrt{3^2+2^2}} = \frac{|13|}{\sqrt{13}}$$

$d(T, q) = \sqrt{13}$

$P_{\square} = d^2 = \sqrt{13}^2$

$\boxed{P_{\square} = 13}$

Napiši jednadžbu pravca paralelnog s pravcem $3x + 4y - 11 = 0$ i od njega udaljenog za $d = 5$.

$p \ldots \; 3x+4y-11=0 \longrightarrow y = -\frac{3}{4}x + \frac{11}{4}$

$d = 5$

$p \parallel q$

$q \ldots ?$

$k_p = k_q = -\frac{3}{4}$

$q \ldots \; y = -\frac{3}{4}x + l$

$q \ldots \; \frac{3}{4}x + y - l = 0$

$T_p(x_T, y_T)$ $\quad x_T = 4$

$y = -\frac{3}{4} \cdot 4 + \frac{11}{4}$

$y = -3 + \frac{11}{4}$

$y = -\frac{1}{4}$

$T_p\left(4, -\frac{1}{4}\right)$

$$d(T, q) = \frac{\left|\frac{3}{4} \cdot 4 + 1 \cdot \left(-\frac{1}{4}\right) - l\right|}{\sqrt{\left(\frac{3}{4}\right)^2 + 1^2}}$$

$$5 = \frac{\left|\frac{11}{4} - l\right|}{\frac{5}{4}}$$

$$\left|\frac{11}{4} - l\right| = \frac{25}{4}$$

$\frac{11}{4} - l = \frac{25}{4}$ $\qquad$ $\frac{11}{4} - l = -\frac{25}{4}$

$l = -\frac{7}{2}$ $\qquad$ $l = 9$

$q \ldots \; \boxed{y = -\frac{3}{4}x - \frac{7}{2}}$ $\qquad$ $q \ldots \; \boxed{y = -\frac{3}{4}x + 9}$

Odredi simetrale kutova što ih zatvaraju pravci $x - 3y + 11 = 0$ i $2x + 6y + 7 = 0$.

$$x - 3y + 11 = 0$$
$$2x + 6y + 7 = 0$$

$$\frac{|1 \cdot x + (-3)y + 11|}{\sqrt{1^2 + (-3)^2}} = \frac{|2 \cdot x + 6 \cdot y + 7|}{\sqrt{2^2 + 6^2}}$$

$$\frac{|x - 3y + 11|}{\cancel{\sqrt{10}}} = \frac{|2x + 6y + 7|}{2\cancel{\sqrt{10}}}$$

$$2 \cdot |x - 3y + 11| = |2x + 6y + 7|$$

$$2 \cdot (x - 3y + 11) = 2x + 6y + 7$$
$$\cancel{2x} - 6y + 22 = \cancel{2x} + 6y + 7$$
$$12y = 15$$
$$y = \frac{5}{4}$$

$$-2(x - 3y + 11) = 2x + 6y + 7$$
$$-2x \cancel{+ 6y} - 22 = 2x \cancel{+ 6y} + 7$$
$$4x = -29$$
$$x = -\frac{29}{4}$$