[MAA 2.6] TRANSFORMATIONS

<u>SOLUTIONS</u>

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O. Practice questions

1.

$y = f(x) + 5$	(1, 5.5)	$y = f(x+5)$	(-4, 0.5)
$y = f(x) - 5$	(1, -4.5)	$y = f(x-5)$	(6, 0.5)
$y = 5f(x)$	(1, 2.5)	$y = f(5x)$	(0.2, 0.5)
$y = f(x)/5$	(1, 0.1)	$y = f(x/5)$	(5, 0.5)
$y = -f(x)$	(1, -0.5)	$y = f(-x)$	(-1, 0.5)

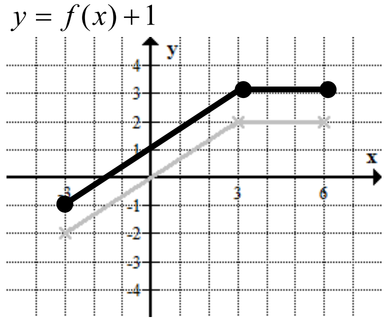
2. (a)

$y = f(x) + 3$	(-1, 6)	$y = f(x+3)$	(-4, 3)
$y = f(x) - 3$	(-1, 0)	$y = f(x-3)$	(2, 3)
$y = 3f(x)$	(-1, 9)	$y = f(3x)$	(-1/3, 3)
$y = f(x)/3$	(-1, 1)	$y = f(x/3)$	(-3, 3)
$y = -f(x)$	(-1, -3)	$y = f(-x)$	(1, 3)

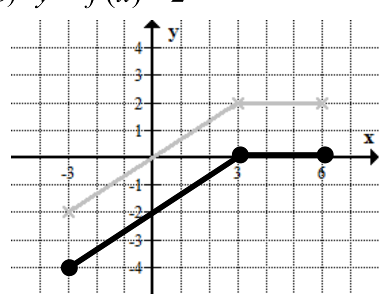
(b)

$f(x)$	(-1, 3)	
$2f(x)$	(-1, 6)	
$2f(x\text{-}3)$	(2, 6)	
$2f(x\text{-}3)+4$	(2, 10)	Hence the corresponding point is (2,10)

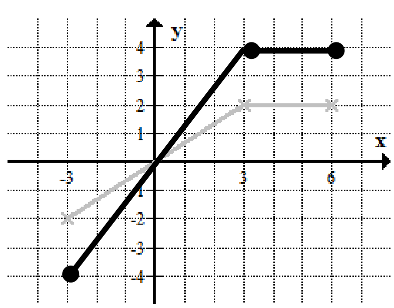
3. (a) $y = f(x) + 1$



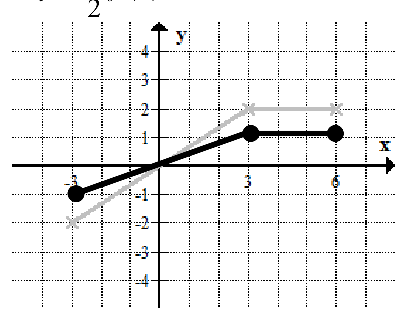
(b) $y = f(x) - 2$



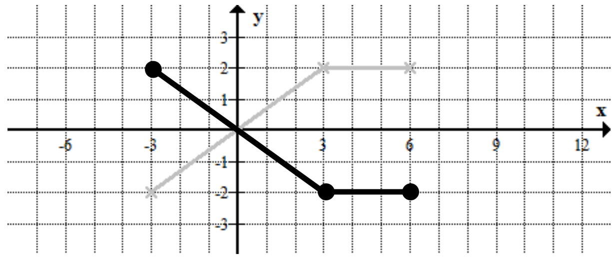
(c) $y = 2f(x)$



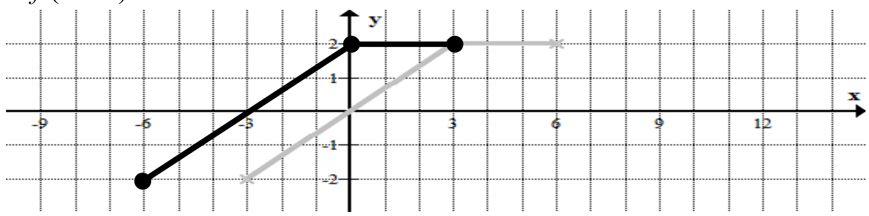
(d) $y = \frac{1}{2} f(x)$



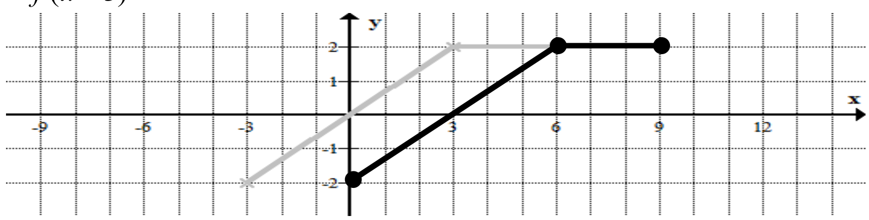
(e) $y = -f(x)$



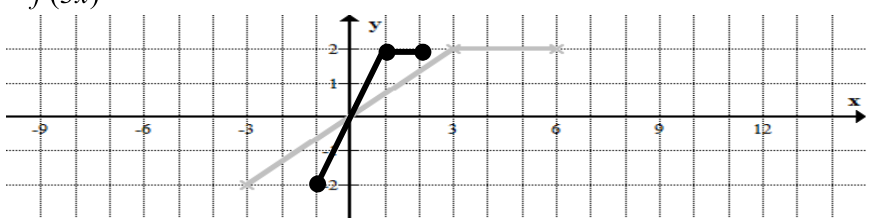
(f) $y = f(x+3)$



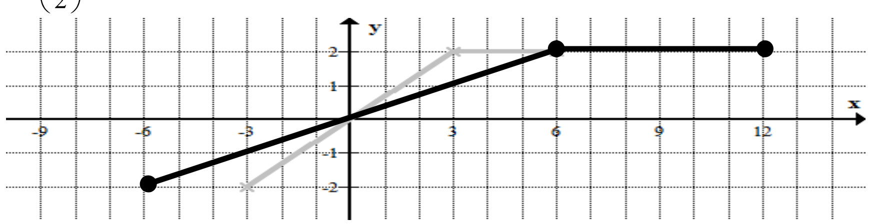
(g) $y = f(x-3)$



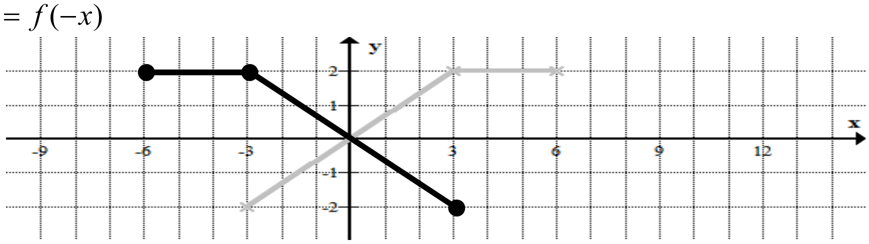
(h) $y = f(3x)$



(i) $y = f\left(\frac{x}{2}\right)$



(j) $y = f(-x)$



4. (a) $-f(x-2)+5$

$f(x)$	original
$-f(x)$	reflection in x-axis
$-f(x-2)$	horizontal translation 2 units to the right
$-f(x-2)+5$	vertical translation 5 units up

(b) $-3f(x+2)-1$

$f(x)$	original
$-f(x)$	reflection in x-axis
$-3f(x)$	vertical stretch with s.f. 3
$-3f(x+2)$	horizontal translation 2 units to the left
$-3f(x+2)-1$	vertical translation 1 unit down

(c) $f(2x-10)$

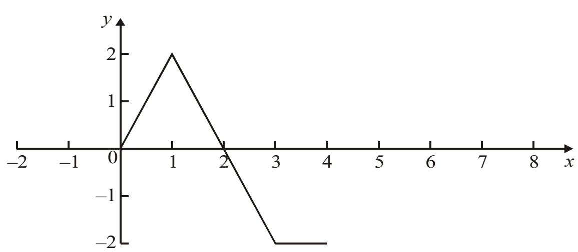
$f(x)$	original
$f(x-10)$	horizontal translation 10 units to the right
$f(2x-10)$	horizontal stretch with s.f. 1/2 (i.e. shrink)

(d) $f(2(x-5))$

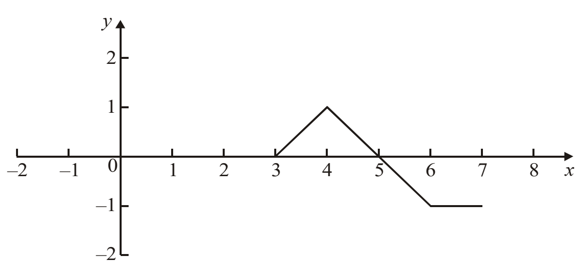
$f(x)$	original
$f(2x)$	horizontal stretch with s.f. 1/2 (i.e. shrink)
$f(2(x-5))$	horizontal translation 5 units to the right

A. Exam style questions (SHORT)

5. (a) (i)



(ii)



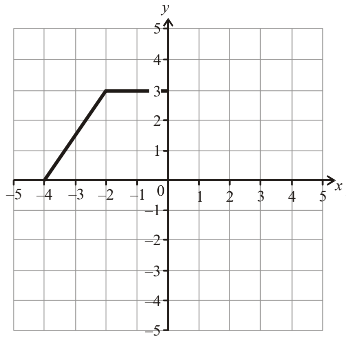
(b) A′ (3, 2) (Accept $x=3$, $y=2$)

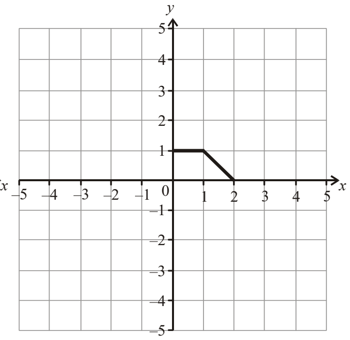
6. (a) (I) D (ii) C (iii) A

(b) B: $f(x)+2$ E: $f(x-2)$

7. (a) (i) 1 (ii) 0.5

(b)

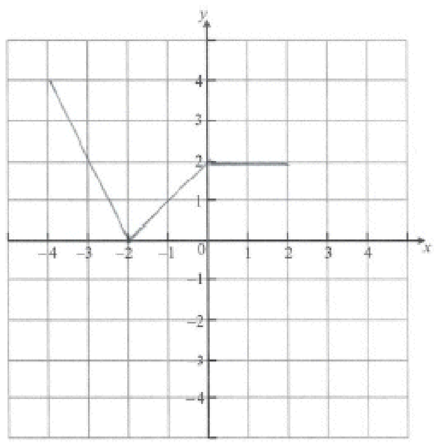




(c)

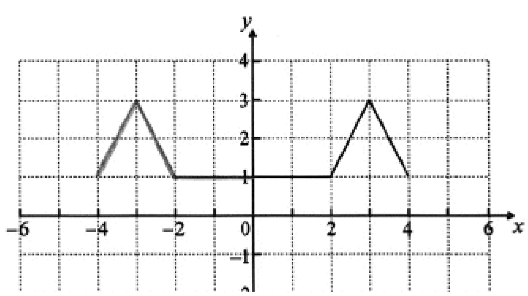
	$y = f(x)$	$y = 3f(-x)$	$y = f(2x)$.
Domain	$0 \le x \le 4$	$-4 \le x \le 0$	$0 \le x \le 2$
Range	$0 \le y \le 1$	$0 \le y \le 3$	$0 \le y \le 1$

8. (a)



(b) $x = 3 + 1$, $y = \frac{1}{2} \times 2$ P is (4, 1)

9. (a)



(b)

Description of transformation	Diagram letter
Horizontal stretch with scale factor 1.5	**C**
Maps f to $f(x) + 1$	**D**

(c) translation (move/shift/slide *etc*.) 6 units to the left and 2 units down

10. (a) By GDC the coordinates are (-1,1.66) [or $\left(-1, \frac{5}{3}\right)$]

[Notice: it can also be found by using derivatives later on]

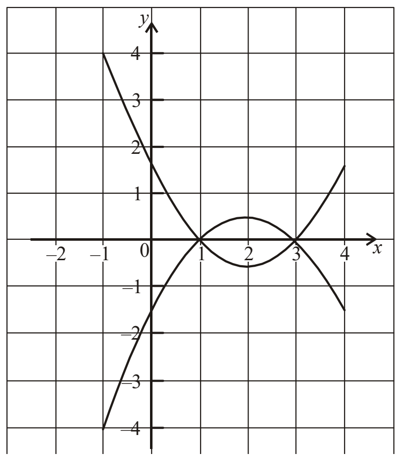
(b) (i) (–3, –9)

(ii) (1, –4)

(iii) reflection gives (3, 9)

stretch gives $\left(\frac{3}{2}, 9\right)$

11. (a)



Note: *reflection in x-axis, correct vertex and all intercepts approximately correct.*

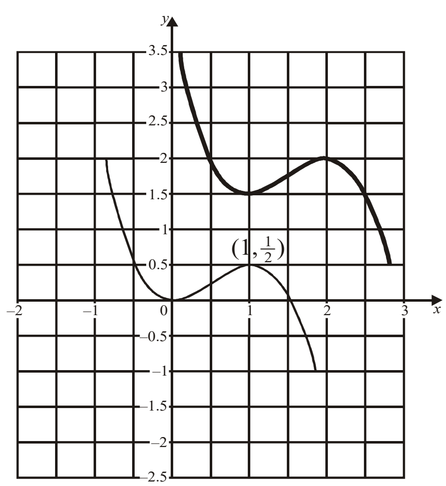
(b) (i) $g(-3) = f(0)$ $f(0) = -1.5$

(ii) translation (accept shift, slide, *etc.*) of $\begin{pmatrix}-3\\0\end{pmatrix}$

(c)

	$y = f(x)$	$y = -f(x)$	$y = f(x+3)$.
Domain	$-1 \le x \le 4$	$-1 \le x \le 4$	$-4 \le x \le 1$
Range	$-4 \le y \le 0.5$	$0.5 \le y \le 4$	$-4 \le y \le 0.5$

12. (a)



(b) Minimum: $\left(1, \frac{3}{2}\right)$ Maximum: (2, 2)

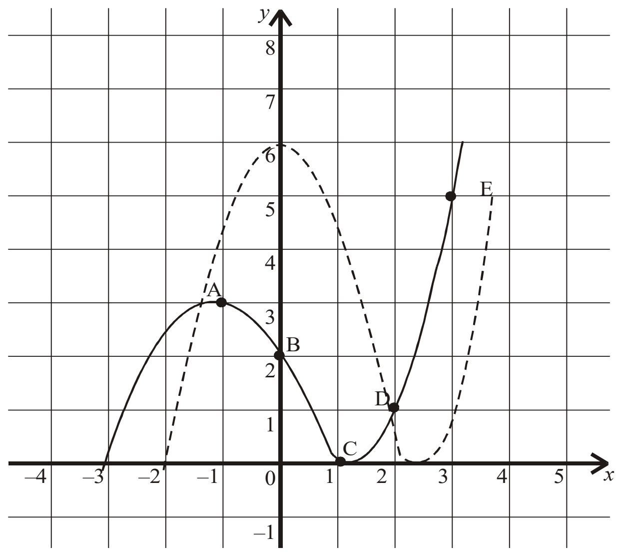
13. (a) $g(x) = 2f(x-1)$

x	0	1	2	3
$x-1$	-1	0	1	2
$f(x-1)$	3	2	0	1

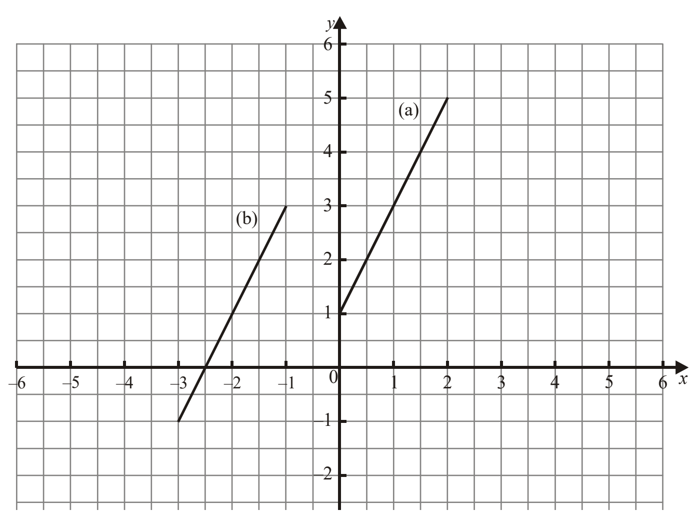
$g(0) = 2f(-1) = 6$ $g(1) = 2f(0) = 4$

$g(2) = 2f(1) = 0$ $g(3) = 2f(2) = 2$

(b) Graph passing through (0, 6), (1, 4), (2, 0), (3, 2)



14. (a)



(b) $g(-3) = -1$ $g(-1) = 3$ Line joining $g(-3)$ and $g(-1)$ drawn

(c)

	$y = f(x)$	$y = g(x)$
Domain	$0 \le x \le 2$	$-3 \le x \le -1$
Range	$1 \le y \le 5$	$-1 \le y \le 3$

15. (a) $y = (x-1)^2$

$y = 4(x-1)^2$

$y = 4(x-1)^2 + 3$

(b)

$y = x^2$	(0,0)
$y = (x-1)^2$	(1,0)
$y = 4(x-1)^2$	(1,0)
$y = 4(x-1)^2+3$	(1,1)

16. (a) in any order

translated 1 unit to the right

stretched vertically by factor 2

(b) **METHOD 1**

Finding coordinates of image on g

$(-1, 1) \rightarrow (-1 + 1, 2 \times 1) = (0, 2)$

P is (3, 0)

METHOD 2

$h(x) = 2(x-4)^2 - 2$

P is (3, 0)

17. (a) $(1, -2)$

(b) $g(x) = 3(x-1)^2 - 2$ (accept $p = 1$, $q = -2$)

(c) (1, 2)

18. (a) $g(x) = -(x-3)^2 - 4$, therefore the maximum point is $(3, -4)$

(b) $f(x)$ is mapped onto $g(x)$ by a reflection in the x-axis followed by the translation $\begin{pmatrix} 3 \\ -4 \end{pmatrix}$.

19. (a) the vertex is at (3, 5)

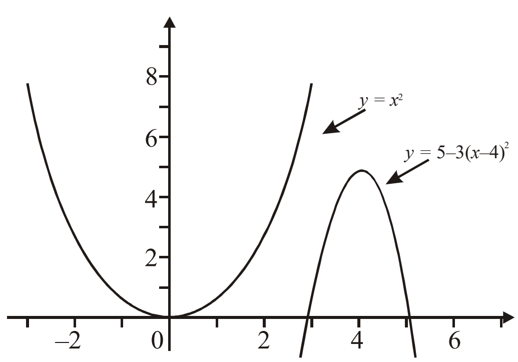
$y = 2(x-3)^2 + 5$

(b) (i) $k = 2$

(ii) $p = 3$

(iii) $q = 5$

20.



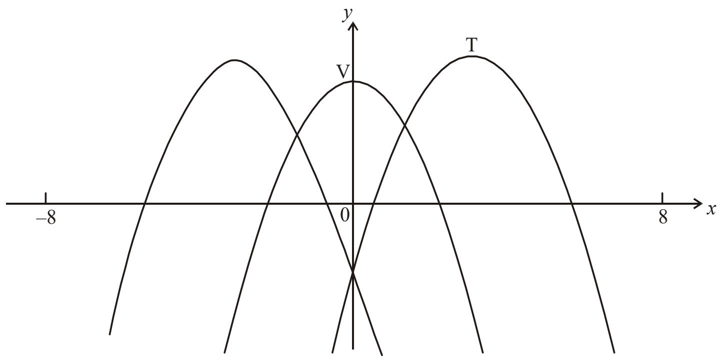
$q = 5$

$k = 3, p = 4$

21. (a) (i) $h = 3$ (ii) $k = 1$

(b) $g(x) = f(x - 3) + 1$, $5 - (x - 3)^2 + 1$, $6 - (x - 3)^2$, $-x^2 + 6x - 3$

(c)



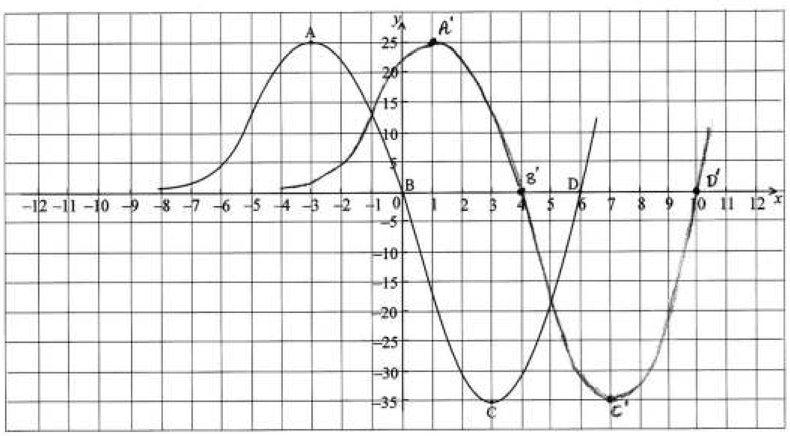
22. (a) By completing the square or finding the vertex (2,1)
$3(x - 2)^2 - 1$

(b) **METHOD 1**
Vertex shifted to (5, 4), $g(x) = 3(x - 5)^2 + 4$

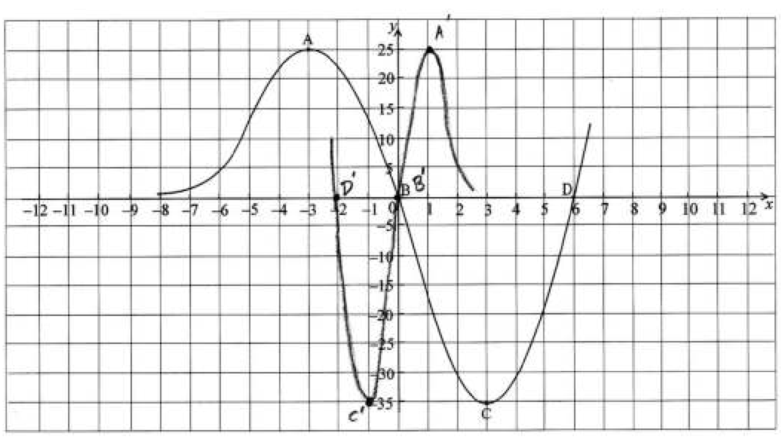
METHOD 2
$g(x) = 3((x - 3)–2)^2 - 1 + 5 = 3(x - 5)^2 + 4$

23.

(a)



(b)



24.

$$\begin{aligned} y &= 2(x-2)^2+4(x-2)+7-1 \\ &= 2(x^2-4x+4)+4x-8+6 \\ &= 2x^2-8x+8+4x-2 \\ \Rightarrow y &= 2x^2-4x+6 \end{aligned}$$

25. $g(x) = f(x-1) - 1 = 2(x-1)^3 - 3(x-1)^2 + (x-1) + 1 - 1 = \ldots = 2x^3 - 9x^2 + 13x - 6$

B. Exam style questions (LONG)

26. (a) $f(x) = 3(x^2+2x+1) - 12 = 3x^2+6x+3-12 = 3x^2+6x-9$

(b) (i) vertex is $(-1, -12)$

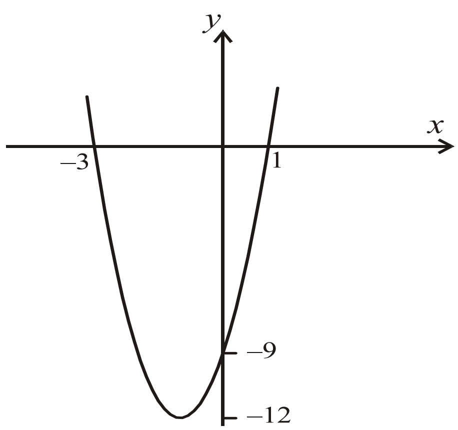
(ii) $x = -1$ (**must** be an equation)

(iii) $(0, -9)$

(iv) $3(x+3)(x-1) = 0$

$(-3, 0)$, $(1, 0)$

(c)



(d) $\begin{pmatrix} p \\ q \end{pmatrix} = \begin{pmatrix} -1 \\ -12 \end{pmatrix}$, $t = 3$ ($p = -1$, $q = -12$, $t = 3$)

27. (a) $(f \circ g)(x) = (x-1)^2 + 4$ ($x^2 - 2x + 5$)

(b) **METHOD 1**

vertex of $f \circ g$ at $(1, 4)$

adding $\begin{pmatrix} 3 \\ -1 \end{pmatrix}$ to the coordinates

vertex of h at $(4, 3)$

METHOD 2

find $h(x) = ((x-3)-1)^2 + 4 - 1$, $h(x) = (f \circ g)(x-3) - 1$

$h(x) = (x-4)^2 + 3$

vertex of h at $(4, 3)$

(c) $h(x) = (x-4)^2 + 3 = x^2 - 8x + 16 + 3 = x^2 - 8x + 19$

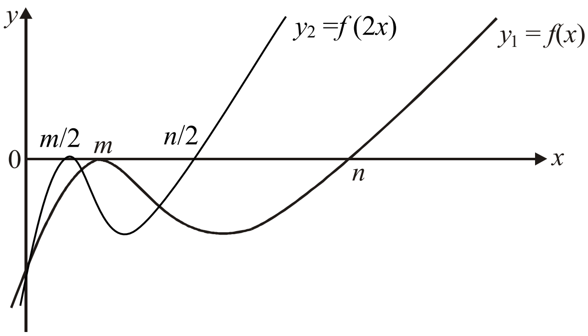
(d) $x^2 - 8x + 19 = 2x - 6$

$x^2 - 10x + 25 = 0 \Leftrightarrow (x-5)^2 = 0$

$x = 5$ ($p = 5$)

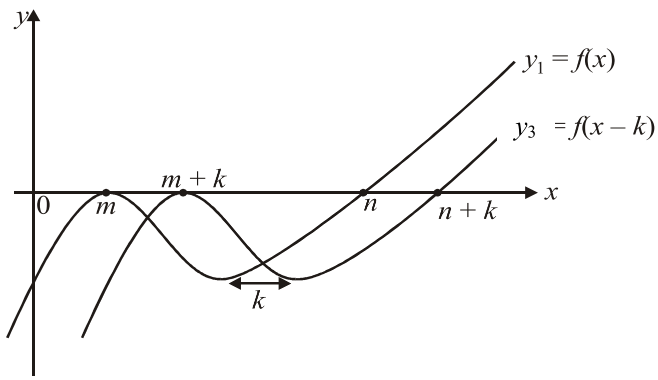
[there is also an alternative method using derivatives which we will study later on]

28. (a)



Notes: *The graph of y_2 is y_1 stretched horizontally by s.f. 1/2 (shrink) points of intersection with the x-axis (m/2, 0) and (n/2, 0)*

(b)



Notes: *The graph of y_3 is y_1 shifted k units to the right. points of intersection with the x-axis (m + k, 0) and (n + k, 0)*

(c) (i) $m/2$, $n/2$

(ii) $m+3$, $n+3$

(iii) $\dfrac{m+3}{2}$, $\dfrac{n+3}{2}$

EITHER by using transformations

OR by substitution

For example, for (iii), the solutions are obtained by letting $2x-3=m$ and $2x-3=n$ so that $x=\dfrac{m+3}{2}$, $x=\dfrac{n+3}{2}$