
Click here to see an entertaining TV interview on Phantom Graphs.

http://www.youtube.com/watch?v=ctZ6gICQ4Pg (http://www.youtube.com/watch?v=ctZ6gICQ4Pg)

Exciting New Development! (2012)

I have recently reproduced all my "PHANTOM GRAPHS" on the excellent AUTOGRAPH system.
This involved a new technique of finding the ACTUAL EQUATIONS of the phantom graphs.
I would like to acknowledge the encouragement given to me by Douglas Butler (Director, iCT Training Centre, Oundle) and in particular the considerable enthusiastic expertise of Simon Woodhead (Development Director, Autograph , Eastmond Publishing Ltd.) in helping with technical problems and producing the following links to the website:

"Autograph Activities".

See further resources on Autograph resources website
www.tsm-resources.com/autograph (http://www.tsm-resources.com/autograph)
Just scroll down to my personal section.

NEWSFLASH!!! (2013)

I recently found some very surprising phantoms which occur when a curve does not have any turning points (eg y = x^3 + x).
The phantoms produced are not joined to the basic graph.
This was a totally unexpected development.
I investigated the idea using the graph of y = x^3 + ax and I varied the value of a.
When a is zero, we get the basic curve y = x^3.
When a is negative, we get the usual cubics with 1max and 1 min and 2 phantoms are joined to the curve at each max/min point.
When a is positive, the phantoms become detached from the the curve.
I have explaned it all on the following SCREENCAST VIDEO.
Just click on the link below.

http://screencast.com/t/1ujgXFbDTu (http://screencast.com/t/1ujgXFbDTu)

(It MAY take a few minutes to load. Please be patient, it is worth it!)

POWERPOINT PRESENTATIONS USED IN LECTURES:

If you would like to see the PHANTOM GRAPH theory in detail on PowerPoint, I have attached it in TWO PARTS.

The PowerPoints are easy to follow and do not need a "presenter" to lead you through the theory and examples.

To see PART 1 click **HERE (/uploads/5/4/5/4/5454288/phantom_graphs_full_presentation_with_autograph_part_1__ver_2.ppt)**.

When downloaded the file will be at the bottom left, click on it then press F5 to play.

To see PART 2 click **HERE (/uploads/5/4/5/4/5454288/phantom_graphs_full_presentation_with_autograph_part_2_ver_2.ppt)**.

(/uploads/5/4/5/4/5454288/phantom_graphs_full_presentation_with_autograph_part_2_ver_2.ppt)

When downloaded the file will be at the bottom left, click on it then press F5 to play.

ANOTHER NEWSFLASH!!! (2015)

I have been thinking about graphs such as *y = x^(n)*,
where n = 0, 1, 2, 3, 4... and comparing them with graphs where n takes values between the whole numbers such as *y = x^(2.1), y = x^(3.5), y = x^(4.25)* and wondering why the left hand half of these graphs disappears when the power is not a whole number.

Click on this link for a screencast video explaining the concept: http://screencast.com/t/yKBLECJdukz

(http://screencast.com/t/yKBLECJdukz)

Click HERE (/uploads/5/4/5/4/5454288/1_ppt_graphs_of_the_form_y_=_x^n_version_6_from_n_=_0_to_5.pptx) to see the amazing PowerPoint!

Click HERE (/uploads/5/4/5/4/5454288/0_the_theory_of_phantoms_of_y_=_x^n.docx) to see the theory of the discovery.

After further research, I have extended the theory to include graphs of the form *y = x^(-n)*.

Click on this video: http://screencast.com/t/TiNZ813k

(http://screencast.com/t/TiNZ813k)

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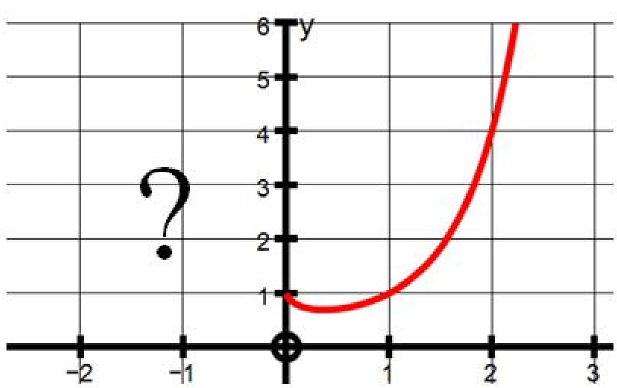
ANOTHER FIRST. (2018)

THE GRAPH OF y = x^x

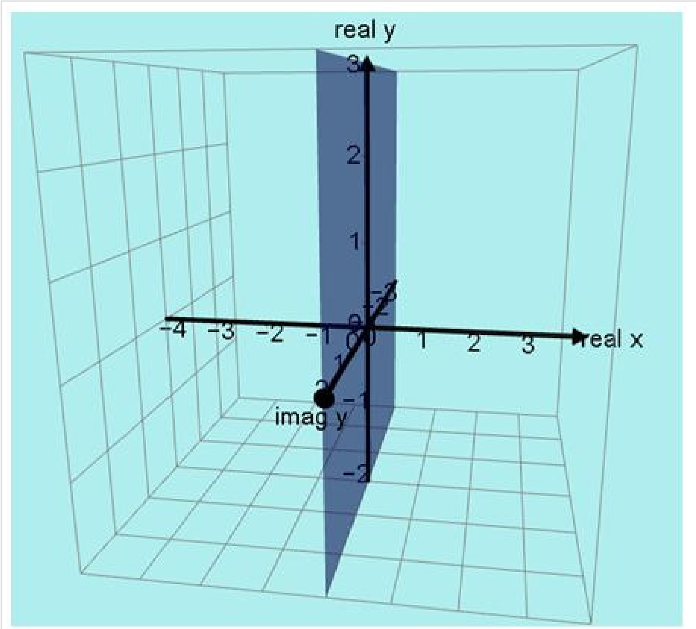
I did a lot of work on this graph in 2015 and I found the graph to be ABSOLUTELY fascinating.

For **x > 0** it seemed to be quite straightforward, it just looks a bit like a very steep exponential curve but the left hand side is where it gets very exciting!

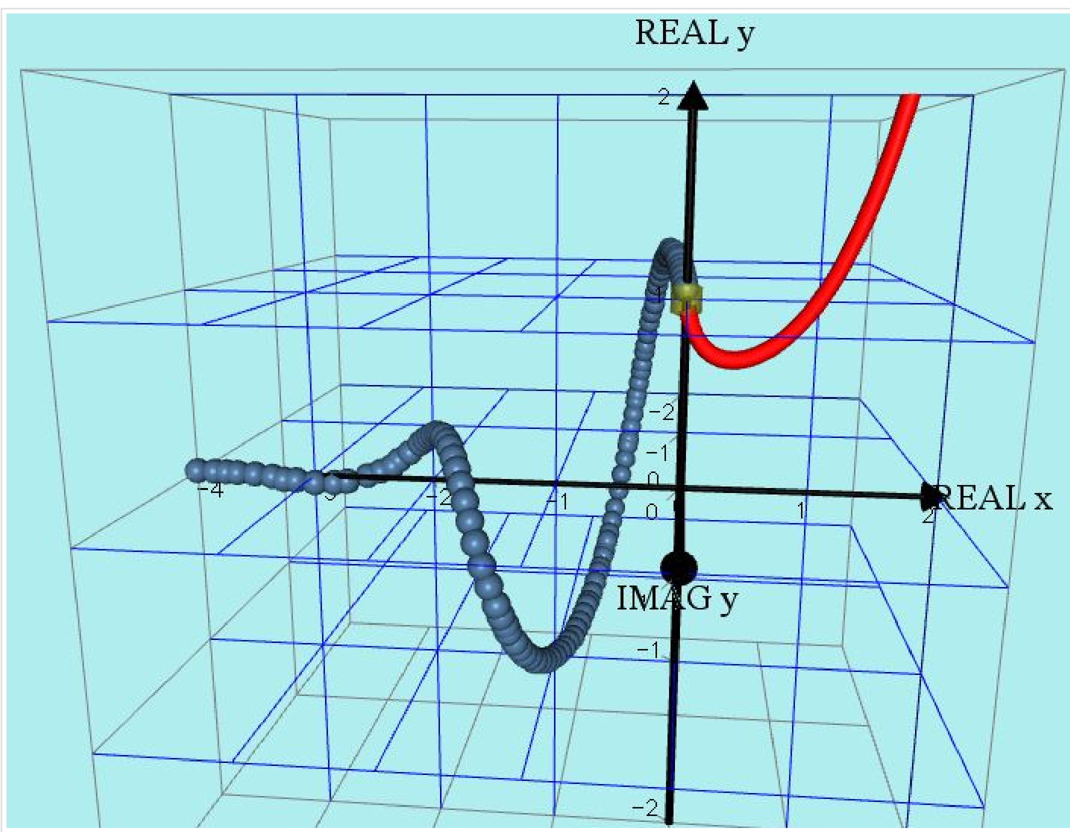


The problem was that quantities such as (-1.6)^(-1.6) have a real part and an imaginary part!
In fact if y = (-1.6)^(-1.6) it works out to be ***0.15 + 0.45i***

Points such as ***x = -1.6, y = 0.15 + 0.45i*** cannot be put on a normal *x, y* graph in 2 dimensions!
We need to put the complex y values on a **complex y plane** not a **y axis**.

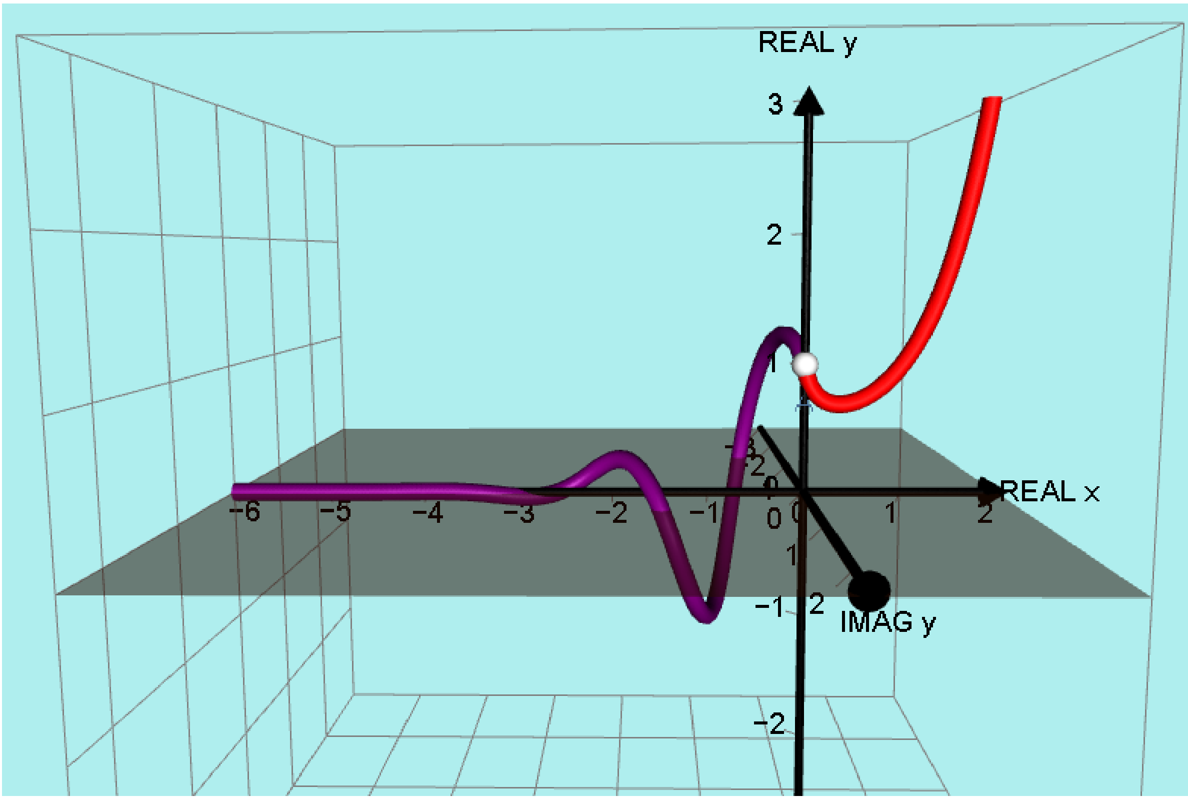


I went to the trouble of calculating plotting many points using negative x values and carefully plotted them on this 3D set of axes…
The result is this FABULOUS SPIRAL!



After further studying I worked out that the actual equation of the spiral was:
$y = |x|^x \cos(\pi x) + |x|^x \sin(\pi x)i$

This was the lovely result: The spiral has a period of 2.



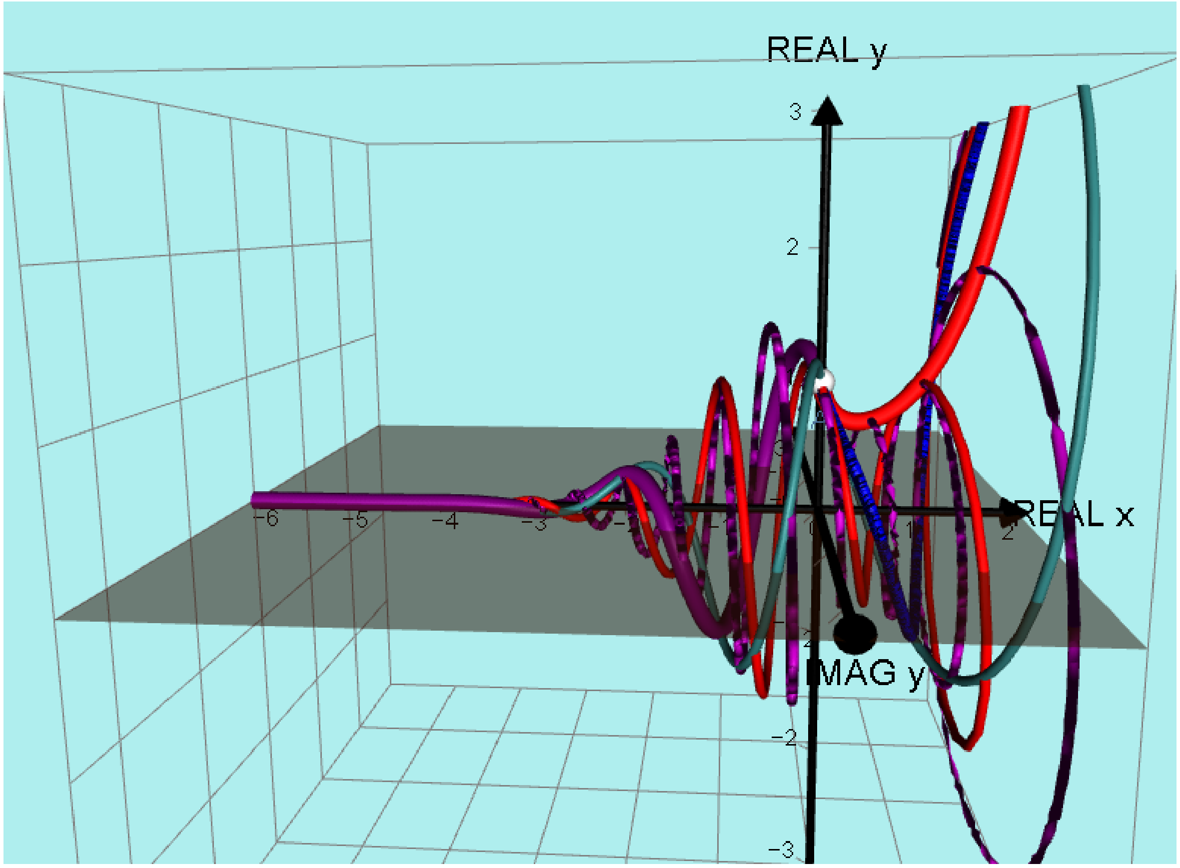
Note: when x = 0 we get y = 0^0 which is undefined so I put a white open "sphere" on that point. It is obvious though that on both sides of x = 0, the curve does approach y = 1.

2018:

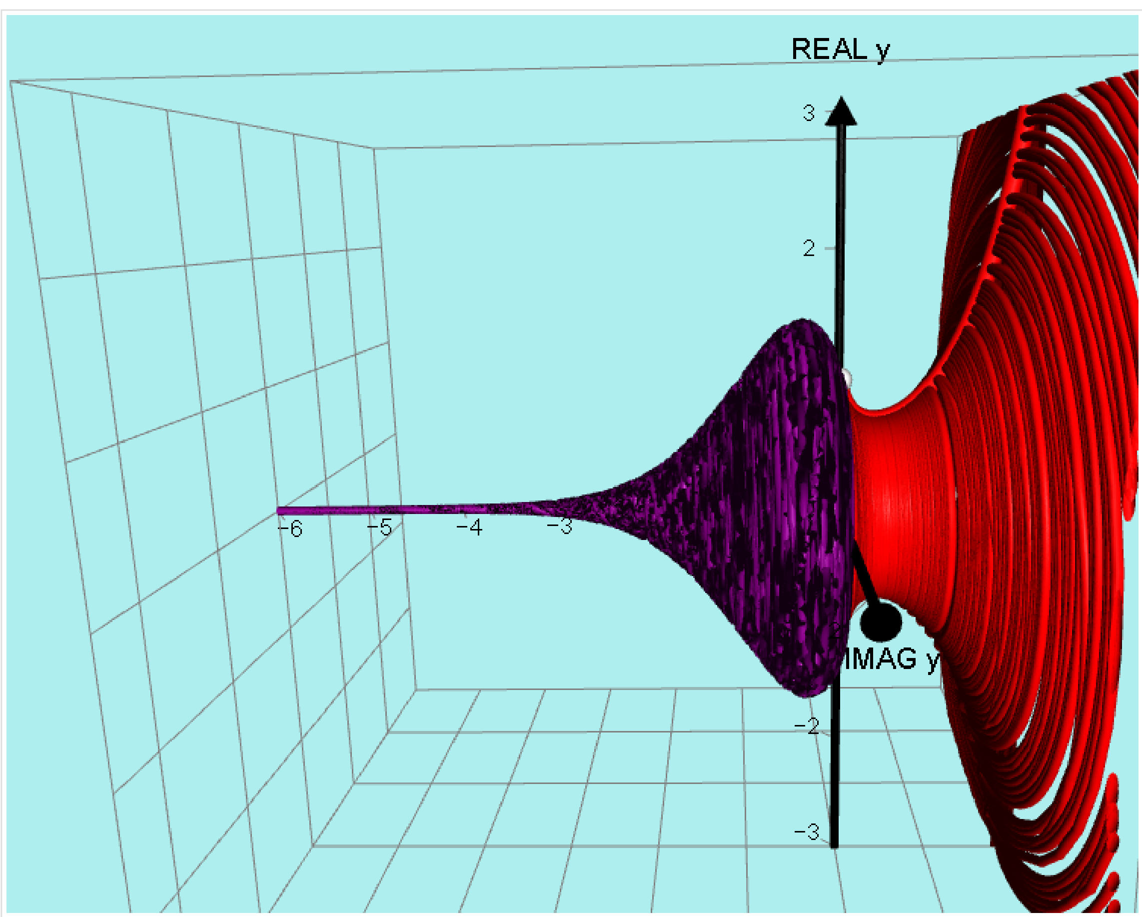
However, I recently became aware, through my friend Robbie Hatley, that the basic equation I used above can have a more general form based on the fact that trig functions repeat at intervals, for example sin(**π) = sin(2π) = sin(nπ)**:

The equation becomes: **y = |x|^x cos(nπx) + |x|^x sin(nπx)*i***

I found it puzzling that all these new graphs will have different periods and after drawing just a few it was **not obvious what was happening**!
It looked like a terrible mess!



However, I carried on and drew many, many graphs and to my delight, I realised they were all part of a clearly defined surface!

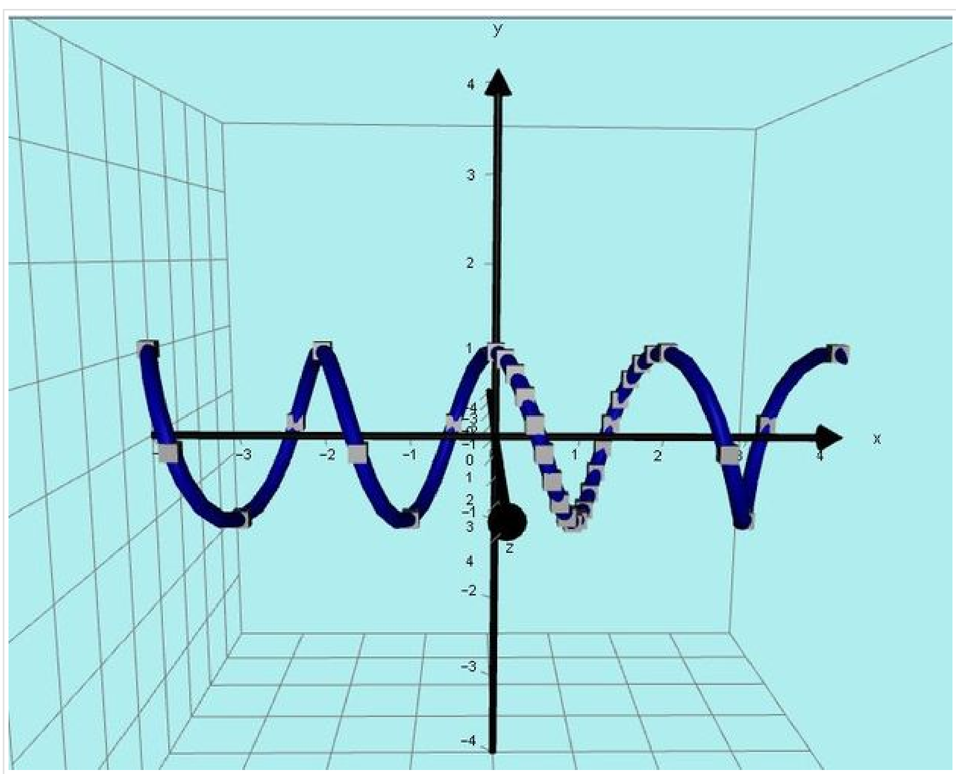


THE GRAPH OF *y = (-1)^x* (2016)

If we just choose INTEGER values of *x* we just get the ***y values +1 and -1***
eg **(0, 1), (1, –1), (2, 1), (3, –1) … and (–1, –1), (–2, 1), (–3, –1)…**
However, if we choose ***x = 0.1*** *we get* ***y = 0.95 + 0.31i***
x = 0.6 we get y = –0.31 + 0.95i
x = 1.2 we get y = –0.81 – 0.59i

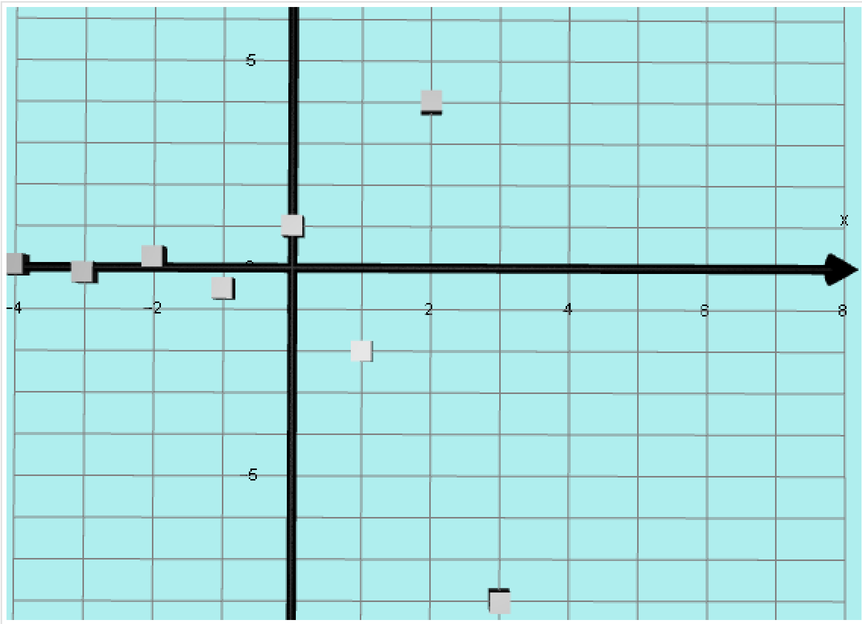
These points have a **real part** and an **imaginary part**.
In order to make sense of this, we need to be able to plot these ***complex y values*** so we need **another axis** besides the ***normal x and y axes***.
I will use only ***real x values*** on the *x axis* and in order to plot points such as
y = 0.95 + 0.31i I will put the **real part** (0.95) on the normal ***y axis***
and the **imaginary part** (0.31*i*) on the ***z axis***.
The DOMAIN of this graph is **all the real numbers** (ie on the real *x* axis).
But instead of a simple ***y AXIS*** we now have a **complex *y PLANE*.**
I plotted several POINTS in this way eg *(0.5, 0 + i) and (0.8, – .81 + .59i)*
and produced this beautiful helix.

y = (-1)^x



THE GRAPH OF *y = (-2)^x* (2016)

If we just choose **INTEGER** values of *x* we get the following points:
eg **(0, 1), (1, –2), (2, 4), (3, –8), (4, –16) … and (–1, – ½), (–2, ¼), (–3, –⅛)…**



However, the graph does not just exist as a set of these isolated points.

If we choose ***x = 0.25*** *we get* ***y = 0.84+ 0.84i***

x = 0.5 we get y = 0 + 1.41i

x = 0.75 we get y = -1.19 + 1.19i

x = 1.25 *we get* ***y = -1.68 - 1.68i***

x = 1.5 we get y = 0 - 2.83i

x = 1.75 we get y = 2.38 - 1.38i

x = -0.25 then y = 0.59 - 0.59i

x = -0.5 then y = 0 - 0.71i

x = -0.75 then y = -0.59 - 0.59i

x = -1.25 then y = -0.3 + 0.3i

x = -1.5 then y = 0 + 0.35i

x = -1.75 then y = 0.2 + 0,2i

etc

These points have a REAL part and an IMAGINARY part.

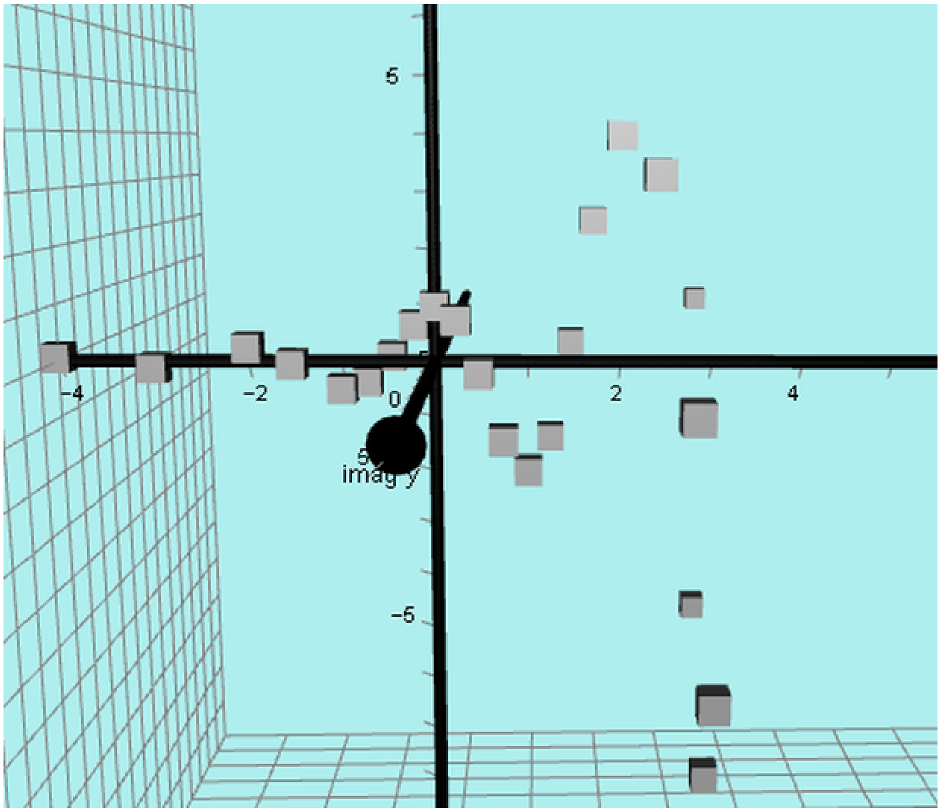
In order to make sense of this, we need to be able to plot these ***complex y values*** so we need **another axis** besides the ***normal x and y axes***.

I will use only ***REAL x VALUES*** on the *x axis* and in order to plot points such as ***y = 0.2 + 0,2i*** I will put the **real part** on the normal ***y axis***
and the **imaginary part** on the ***z axis (imaginary y axis)***, using **Autograph**.

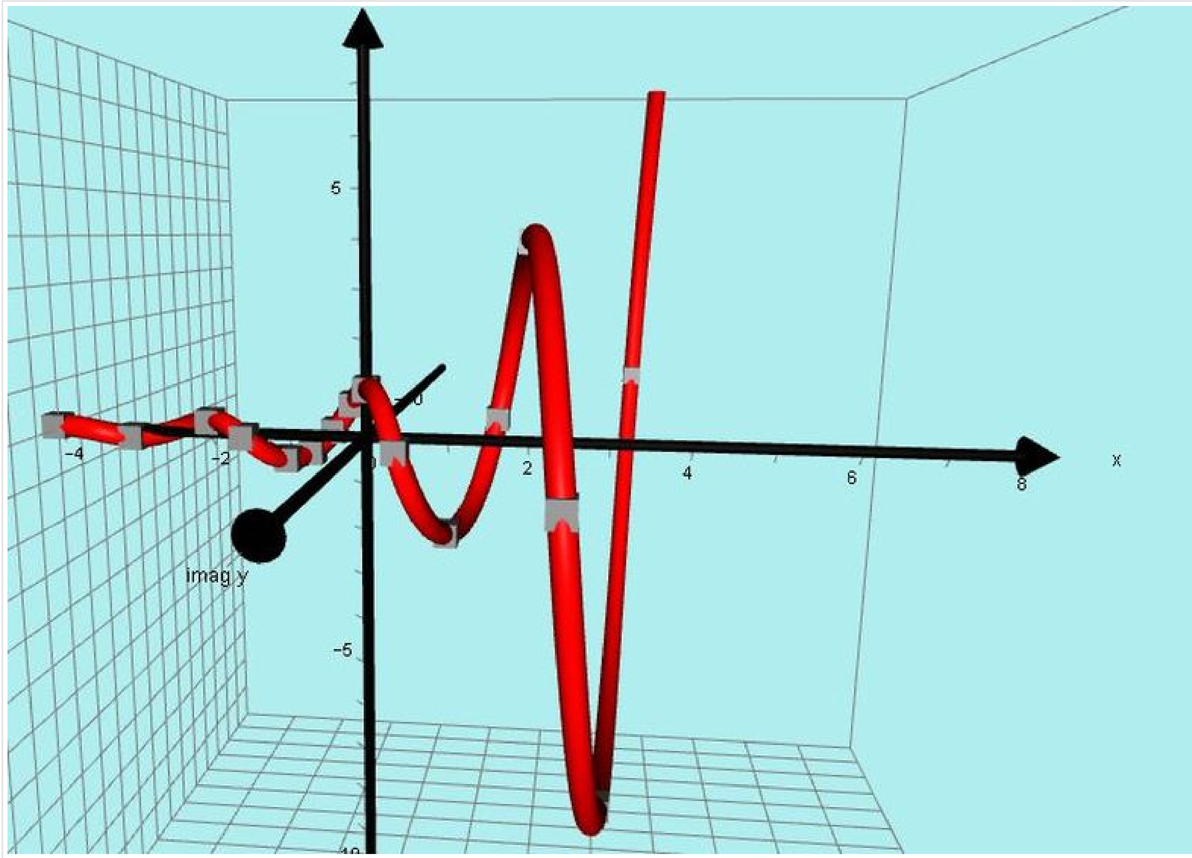
The DOMAIN of this graph is all the real numbers (ie on the real *x* axis).

But instead of a simple ***y AXIS*** we now have a ***complex y PLANE***.

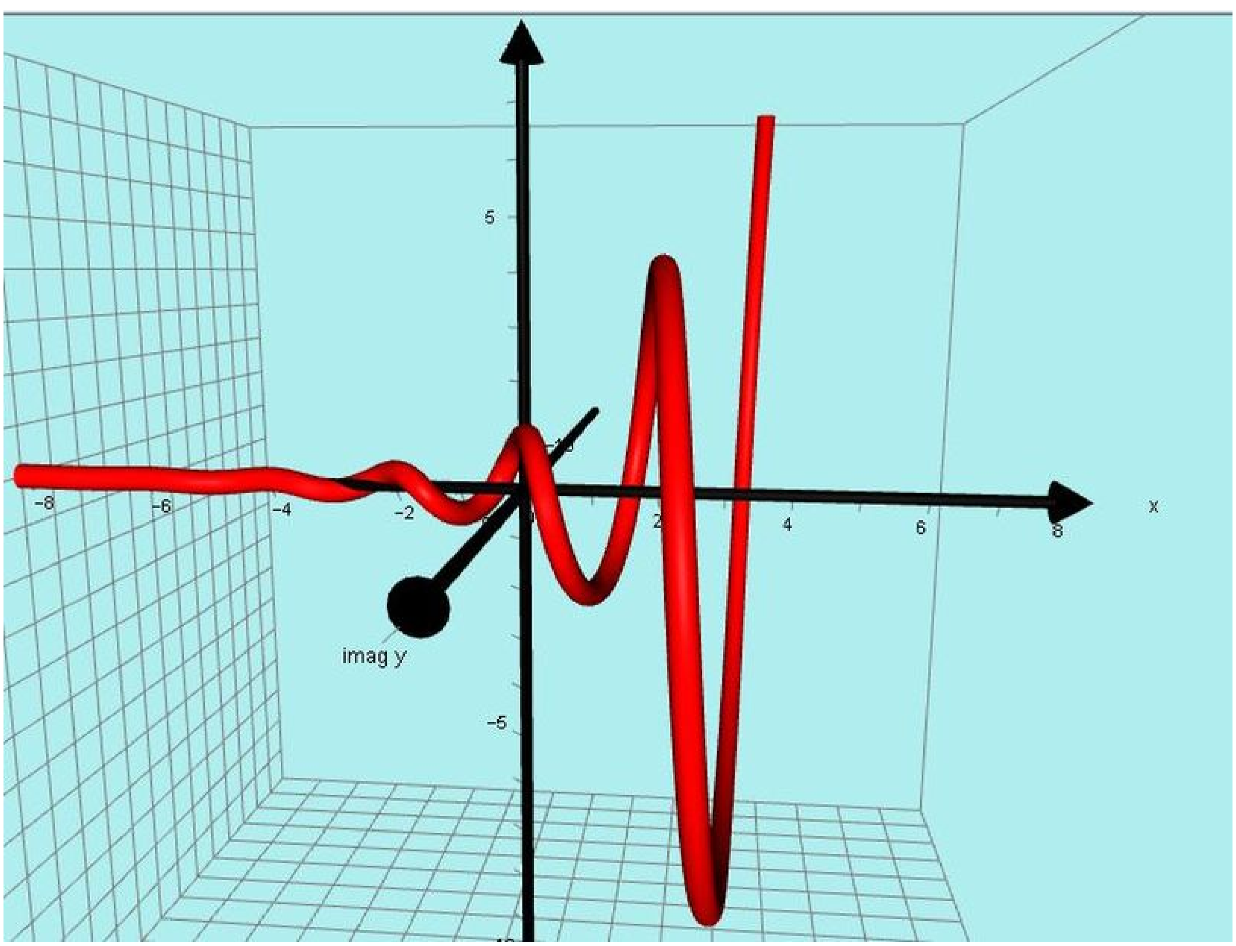
I plotted the POINTS listed above and several more *and produced this amazing spiral!*



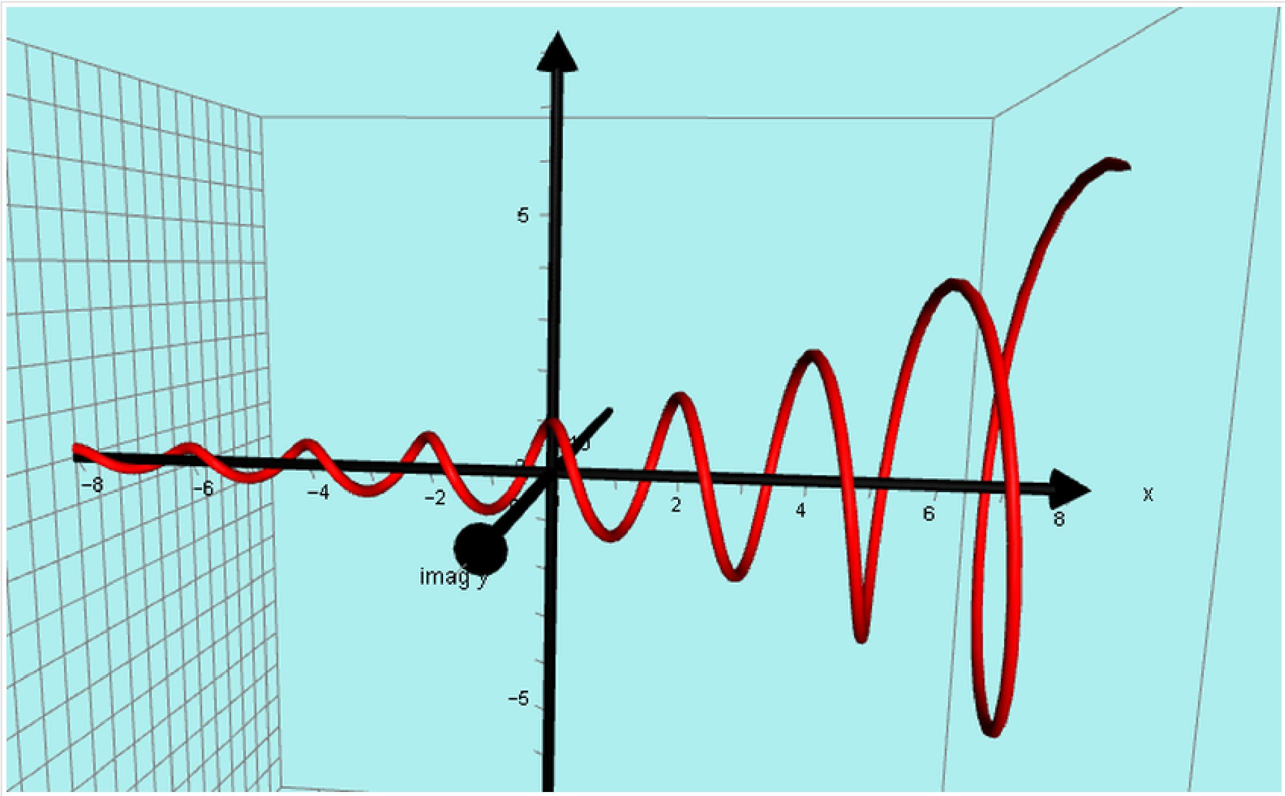
I then found the **equation** of the curve and the shape becomes clearer.



Here is another version without the extra “points” ***y = (−2)^x***



An interesting variation is to change the base of the equation to
y = (-1.25)^x



2016: In response to questions on various internet sites about how to visualize where the complex roots of cubic functions are, I decided to produce the following explicit explanations.

This screencast video will give you a visual demonstration of the theory given below.

http://screencast.com/t/dkAYxFDwH (http://screencast.com/t/dkAYxFDwH)

Let's just consider "cubics" and state what the Fundamental Theorem of Algebra says.

i.e. that any cubic equation of the form: $ax^3 + bx^2 + cx + d = 0$ will have three solutions.

It seems quite reasonable to expect the graph of $y = ax^3 + bx^2 + cx + d$ to cross the x axis 3 times because $y = 0$ exactly 3 times.

This does not seem to be true if 2 of the solutions are complex numbers but in fact it is still true!

We find that some special complex x values will still give us real y values.

Consider the equation $y = x^3 - x^2 + 2 = (x + 1)(x^2 - 2x + 2)$

$$= (x + 1)(\, x - (1 + i)\,)\,(x - (1 - i))$$

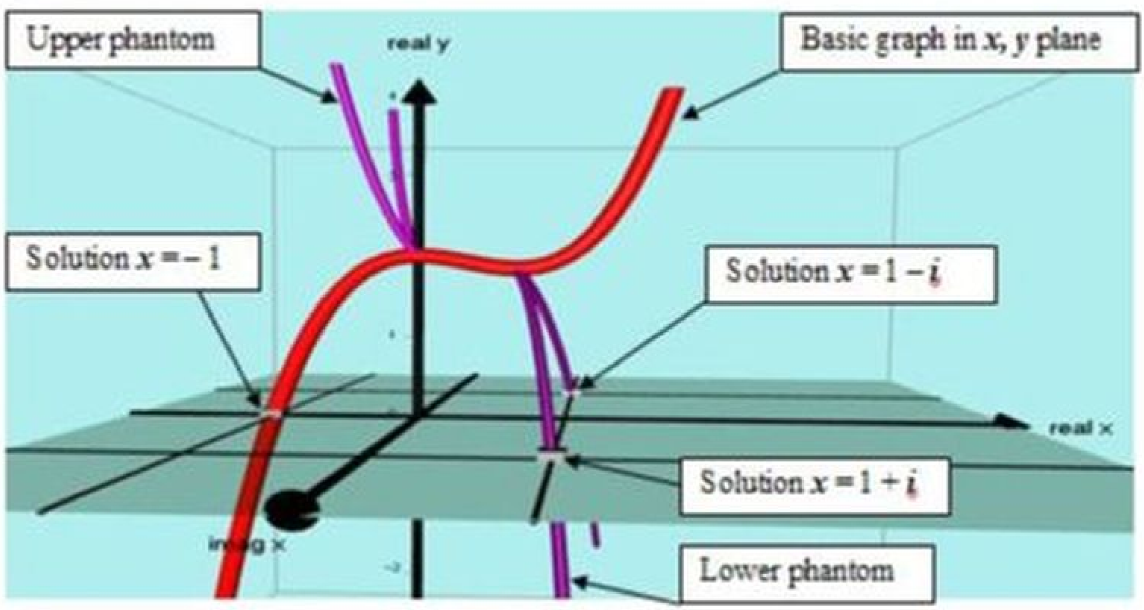
The solutions of $x^3 - x^2 + 2 = 0$ are $x = -1$, $x = 1 + i$ and $x = 1 - i$ so logically the y value of the graph is zero at these 3 places.

The inspirational idea is to create a complex x plane instead of an x axis.

Doing this produces what I call "Phantom Graphs".

These phantom graphs consist of all the complex x values which still have real y values.

In cases like the above equation, these phantom bits are joined to the basic graph at its turning points as you will see on the diagram below.



And we can see that the graph does in fact cross the x plane 3 times at the x values $x = -1$, $x = 1 + i$ and $x = 1 - i$ which are the solutions of the equation $x^3 - x^2 + 2 = 0$

The actual equations of the phantom graphs for the original graph, $y = x^3 - x^2 + 2$ can be obtained as follows:

Recall that I said some special complex x values will still give us real y values. The first step is to allow complex x values by changing x into $x + iz$ where z is the 3rd axis at right angles to the normal x, y plane.
The equation is now $y = (x + iz)^3 - (x + iz)^2 + 2$

Expanding and separating into real and imaginary parts, we get:
$y = \left[(x^3 - 3xz^2) - (x^2 - z^2) + 2\right] + i\left[(3x^2z - z^3) - 2xz\right]$ —— *Equation A*

but we only want REAL y values so the imaginary part $3x^2z - z^3 - 2xz = 0$
Factorising this expression, we get: $z(3x^2 - z^2 - 2x) = 0$ this means that either $z = 0$
(which means Equation A just becomes the basic equation $y = x^3 - x^2 + 2$)

or $z^2 = 3x^2 - 2x$
and on substituting this into Equation A we get:

$y = (x^3 - 3x(3x^2 - 2x)) - (x^2 - (3x^2 - 2x)) + 2$ *together with* $z = \pm(3x^2 - 2x)^{1/2}$
and these two equations give coordinates x, y and z for the two phantom graphs.

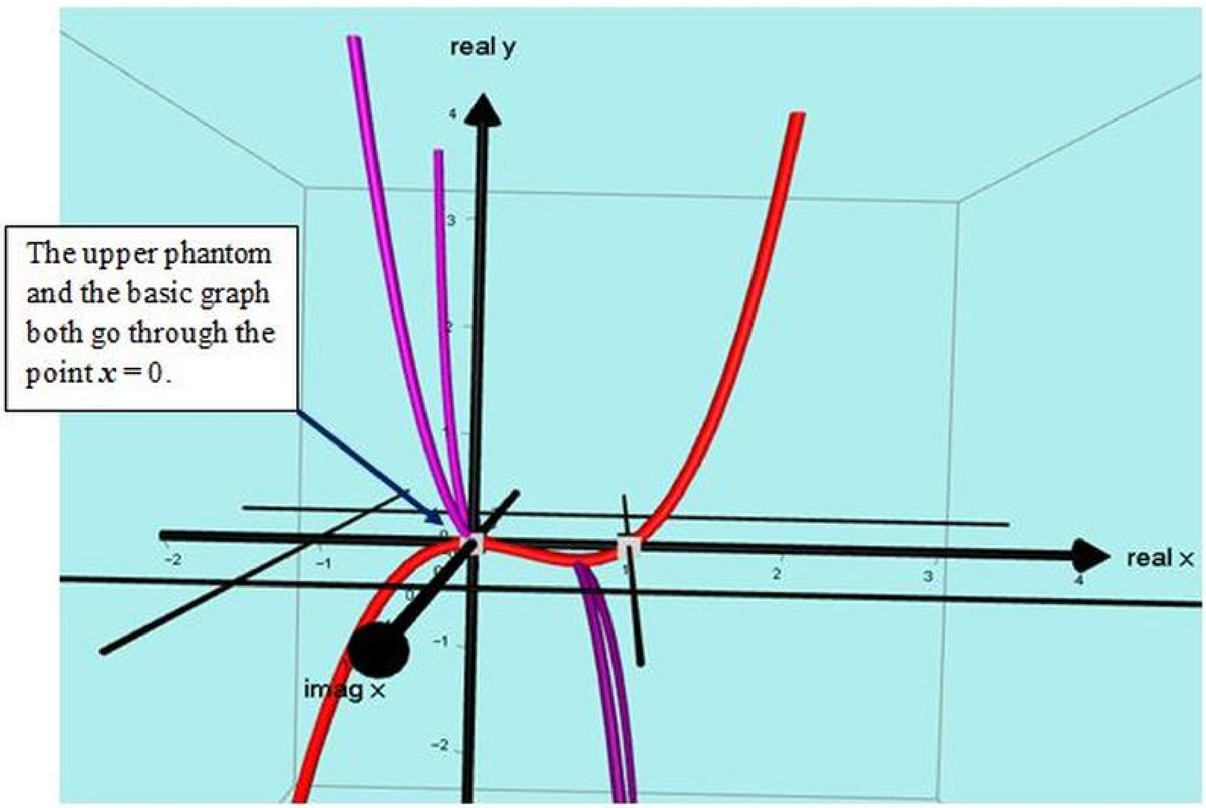
I have used the Autograph program to draw these 3D graphs and to do this the equations needed to be "parametrized" as follows:
Basic graph $y = t^3 - t^2 + 2,\ x = t,\ z = 0$
Phantoms $y = (t^3 - 3t(3t^2 - 2t)) - (t^2 - (3t^2 - 2t)) + 2,\ x = t,\ z = \pm(3t^2 - 2t)^{1/2}$

Special interesting point:

If we simply translate the graph down 2 units the equation becomes $y = x^3 - x^2 = x^2(x - 1)$ which only crosses the x axis at the points $x = 0$ and $x = 1$ so in this case the equation only has real solutions.

Now this seems to violate the fundamental theorem of algebra but the graph does in fact cross the x axis 3 times!

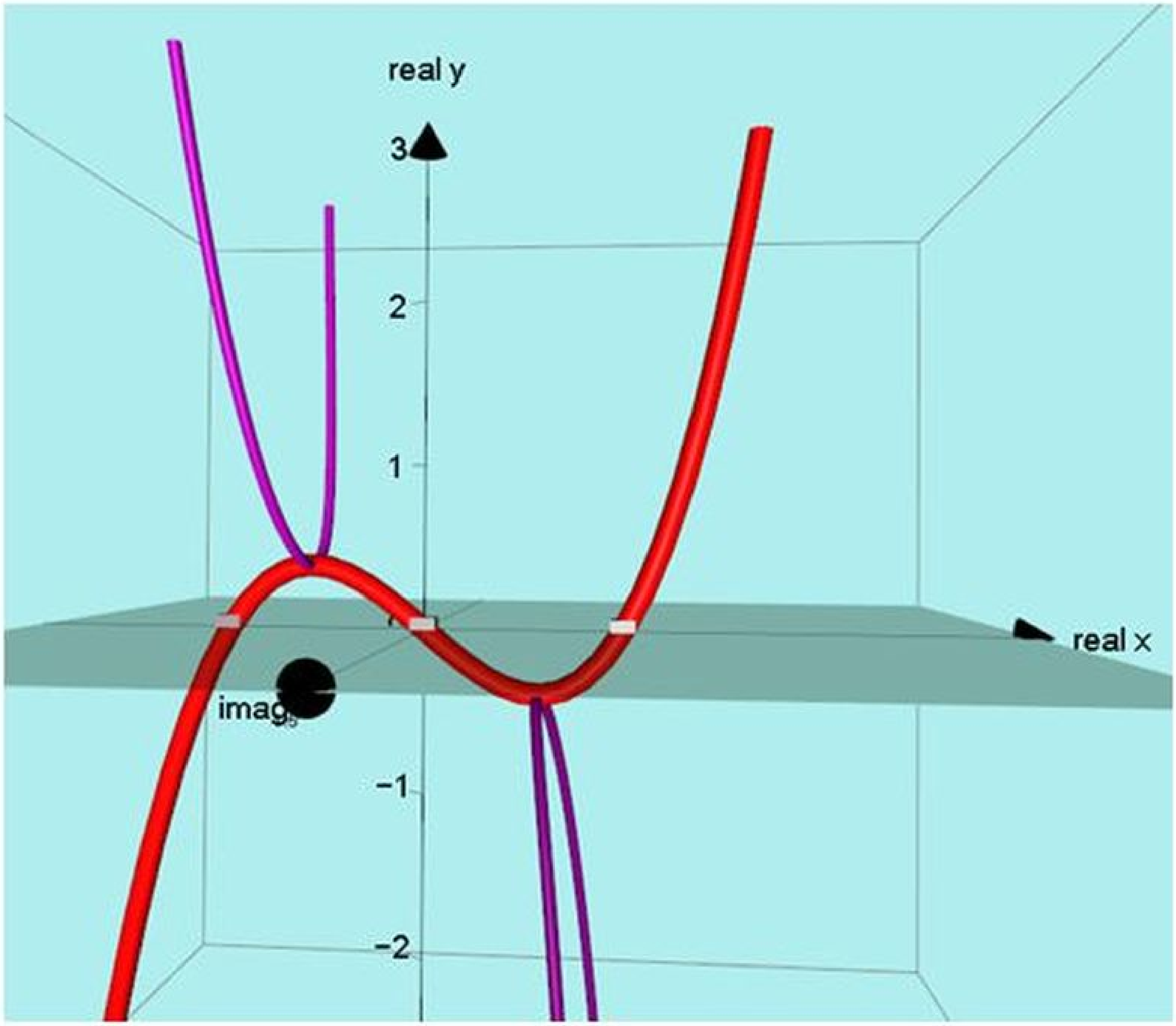
The graph goes once through $x = 1$ and if we consider the point where $x = 0$, the basic graph goes through this point and the upper phantom goes through this point too.



Graph showing the THREE real roots of the equation:

$$y = x^3 - x = x(x^2 - 1) = x(x + 1)(x - 1)$$

(Roots are x = 0, x = –1, and x =1)

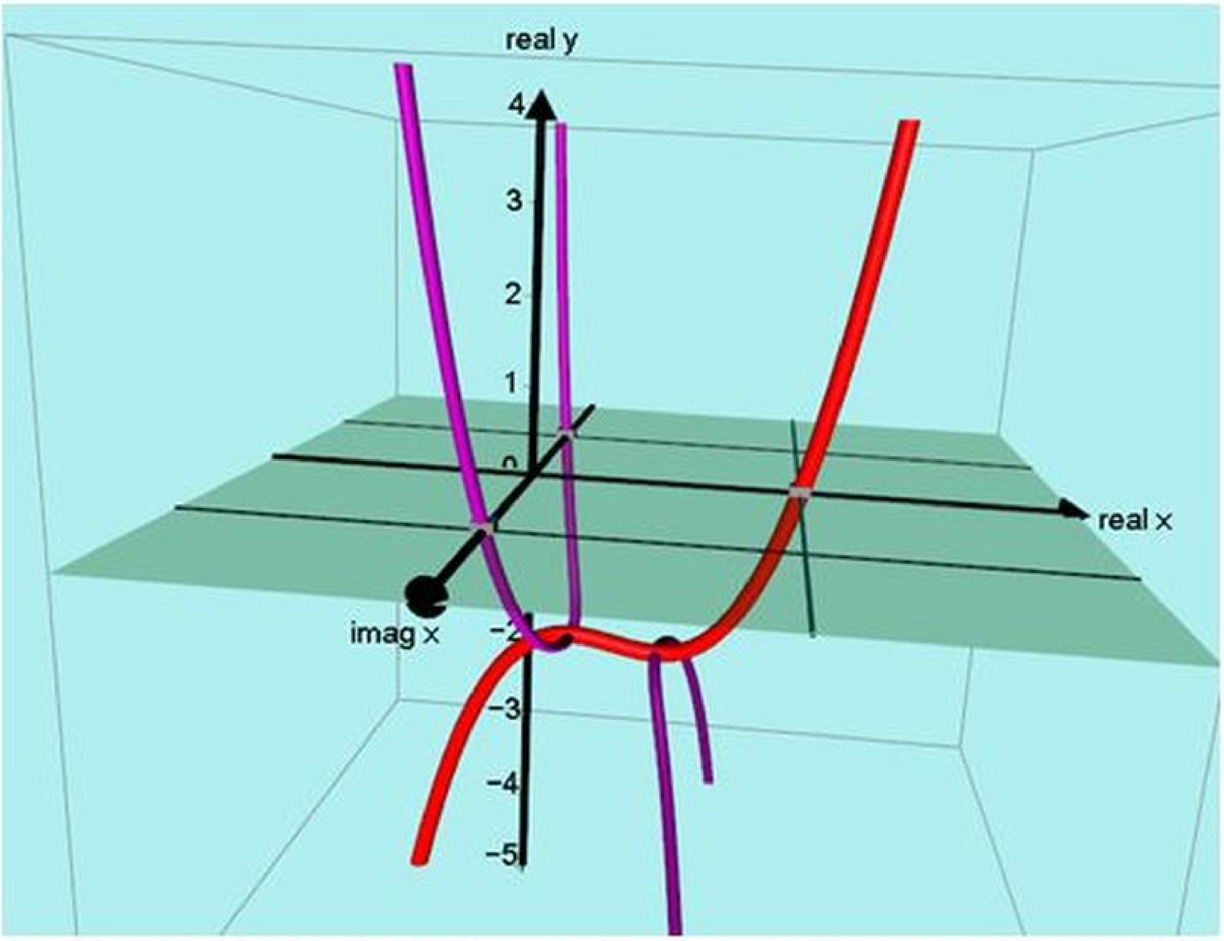


In this case the basic RED curve has two turning points from which the two PURPLE phantoms emerge.
Both the upper and lower phantoms do not cross the x plane so they play no part in the solutions of $x^3 - x = 0$
The basic RED curve crosses the x plane THREE times showing the real solutions to $x^3 - x = 0$ are x = 0, x = –1, and x = 1

Graph showing the one real and two complex roots of the equation:

$$y = x^3 - 2x^2 + x - 2 = (x^2 + 1)(x - 2)$$
$$= (x + i)(x - i)(x - 2)$$

(Roots are $x = i$, $x = -i$ and $x = 2$)



In this case the basic RED curve has two turning points from which the two PURPLE phantoms emerge.

The lower phantom does not cross the x plane so this does not show any solutions to $y = 0$.

The upper phantom crosses the x plane showing the solutions $x = i$ and $x = -i$.

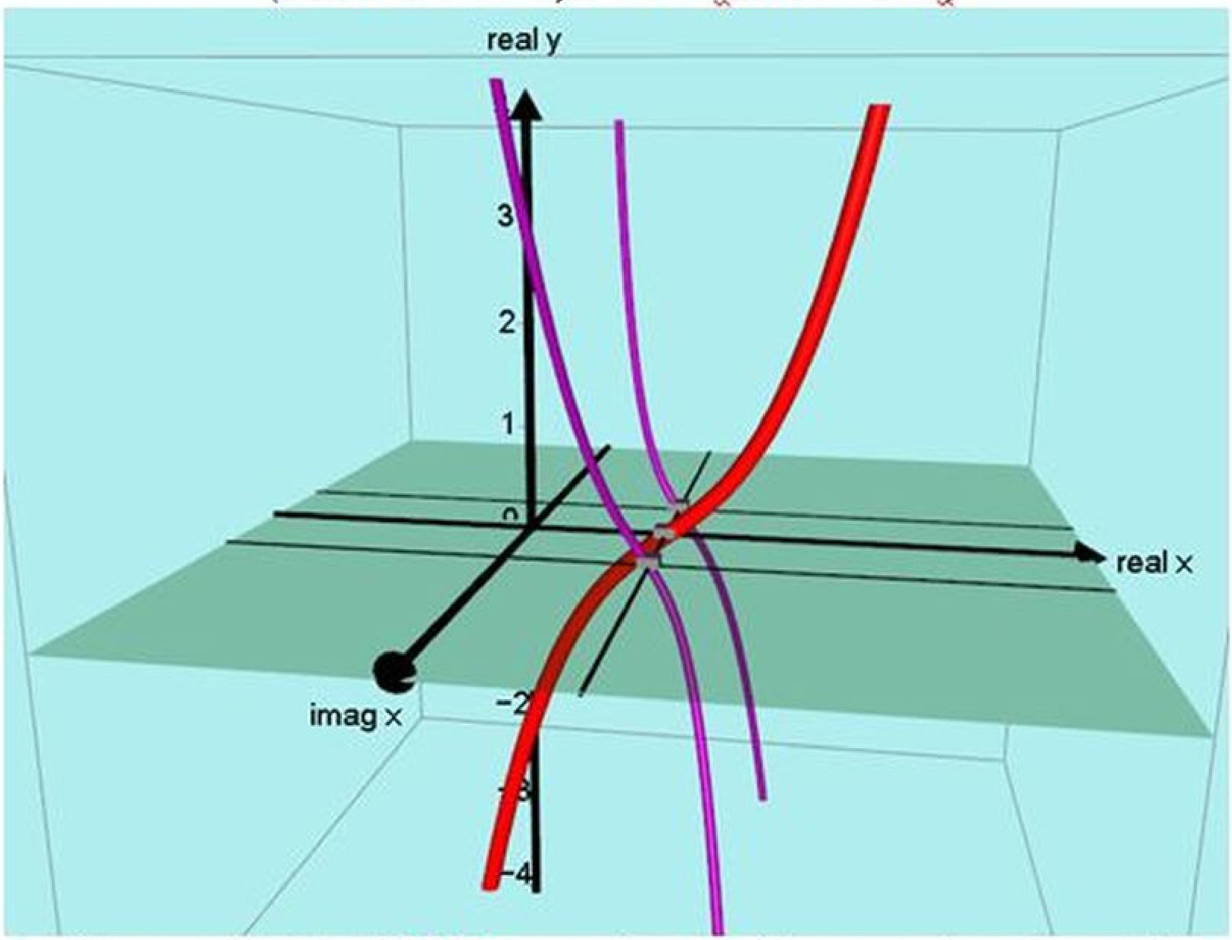
The basic curve crosses the x plane showing the real solution at $x = 2$.

Graph showing the one real and two complex roots of the equation:

$$y = = x^3 - 3x^2 + 4x - 2 = (x - 1)(x^2 - 2x + 2)$$

$$= (x - 1)\ (x - (1 + i))\ (x - (1 - i))$$

(Roots are x = 1, x = 1 + i and x =1 – i)



In this case, the basic RED curve does not have any turning points and the two PURPLE phantoms are not joined to the curve.
The phantom on the left crosses the x plane at x = 1 + i
The phantom on the right crosses the x plane at x = 1 – i
The basic curve crosses the x plane showing the real solution at x = 1.
The unusual thing about this graph is that the real and complex solutions are in a straight line.

SEE VIDEO on INTERSECTION POINTS of CIRCLES with PARABOLAS

http://screencast.com/t/3bdbGbI1u (http://screencast.com/t/3bdbGbI1u)

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