

En los inciso i), ii), iii) y iv) siguientes
a) determine la matriz de transición de B_1 a B_2 .

$$i) B_1 = \{(1, 3), (-2, -2)\}, B_2 = \{(-12, 0), (-4, 4)\}, [u]_{B_2} = (-1, 3)$$

Solución:

$$[B_1/B_2] = \left[\begin{array}{cc|cc} 1 & -2 & -12 & -4 \\ 0 & 4 & 36 & 16 \end{array} \right] \left[\frac{1}{4}f_2 \rightarrow f_2 \right] \sim$$

$$\left[\begin{array}{cc|cc} 1 & -2 & -12 & -4 \\ 0 & 1 & 9 & 4 \end{array} \right] \left[-2f_2 + f_1 \rightarrow f_1 \right] \sim \left[\begin{array}{cc|cc} 1 & 0 & 6 & 4 \\ 0 & 1 & 9 & 4 \end{array} \right]$$

Por lo tanto

$$P_{B_2 \rightarrow B_1} = \begin{bmatrix} 6 & 4 \\ 9 & 4 \end{bmatrix}$$

$$ii) B_1 = \{(2, -2), (6, 3)\}, B_2 = \{(1, 1), (32, 31)\}, [u]_{B_2} = (2, -1)$$

Solución:

$$[B_1/B_2] = \left[\begin{array}{cc|cc} 1 & 32 & 2 & 6 \\ 1 & 31 & -2 & 3 \end{array} \right] \left[\frac{1}{2}f_1 \rightarrow f_1 \right] \sim \left[\begin{array}{cc|cc} 1 & 3 & \frac{1}{2} & 16 \\ -2 & 3 & 1 & 31 \end{array} \right] \left[-2f_1 + f_2 \rightarrow f_2 \right] \sim$$

$$\left[\begin{array}{cc|cc} 1 & 3 & \frac{1}{2} & 16 \\ 0 & 9 & \frac{1}{2} & 63 \end{array} \right] \left[\frac{1}{9}f_2 \rightarrow f_2 \right] \sim \left[\begin{array}{cc|cc} 1 & 3 & \frac{1}{2} & 16 \\ 0 & 1 & \frac{1}{2} & 7 \end{array} \right] \left[-3f_2 + f_1 \rightarrow f_1 \right]$$

Por lo tanto

$$P_{B_2 \rightarrow B_1} = \begin{bmatrix} \frac{1}{9} & -5 \\ \frac{2}{9} & 7 \end{bmatrix}$$

$$iii) B_1 = \{(4, 2, -4), (6, -5, -6), (2, -1, 8)\}, B_2 = \{(1, 0, 4), (4, 2, 8), (2, 5, -2)\}, [u]_{B_2} = (1, -1, 2)$$

Solución:

$$[B_1/B_2] = \left[\begin{array}{ccc|ccc} 4 & 6 & 2 & 1 & 4 & 2 \\ 2 & -5 & -1 & 0 & 2 & 5 \\ -4 & -6 & 8 & 4 & 8 & -2 \end{array} \right] \left[\frac{1}{4}f_1 \rightarrow f_1 \right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{6}{4} & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} \\ 2 & 5 & -1 & 0 & 2 & 5 \\ -4 & -6 & 8 & 4 & 8 & -2 \end{array} \right] \left[\begin{array}{l} -2f_1 + f_2 \rightarrow f_2 \\ -4f_1 + f_3 \rightarrow f_3 \end{array} \right] \sim$$

$$\left[\begin{array}{ccc|ccc} 1 & \frac{6}{4} & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} \\ 0 & -8 & -2 & \frac{1}{2} & 0 & 4 \\ 0 & 0 & 10 & 5 & 12 & 0 \end{array} \right] \left[\begin{array}{l} -\frac{1}{8}f_2 \rightarrow f_2 \\ \frac{1}{10}f_3 \rightarrow f_3 \end{array} \right] \sim$$

$$\begin{aligned} & \left[\begin{array}{ccc|ccc} 1 & \frac{6}{4} & \frac{1}{2} & \frac{1}{4} & 1 & \frac{1}{2} \\ 0 & 1 & \frac{1}{4} & -\frac{1}{16} & 0 & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & \frac{6}{5} & 0 \end{array} \right] \begin{array}{l} [-\frac{1}{2}f_3 + f_1 \rightarrow f_1] \\ [-\frac{1}{4}f_3 + f_2 \rightarrow f_2] \end{array} \sim \\ & \left[\begin{array}{ccc|ccc} 1 & \frac{6}{4} & 0 & 0 & \frac{2}{5} & -\frac{1}{2} \\ 0 & 1 & 0 & -\frac{1}{16} & -\frac{3}{10} & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & \frac{6}{5} & 0 \end{array} \right] [-\frac{6}{4}f_2 + f_1 \rightarrow f_1] \sim \\ & \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \frac{3}{32} & \frac{17}{20} & \frac{5}{4} \\ 0 & 1 & 0 & -\frac{1}{16} & -\frac{3}{10} & -\frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & \frac{6}{5} & 0 \end{array} \right] \end{aligned}$$

Por lo tanto

$$P_{B_2 \rightarrow B_1} = \begin{bmatrix} \frac{3}{32} & \frac{17}{20} & \frac{5}{4} \\ -\frac{1}{16} & -\frac{3}{10} & -\frac{1}{2} \\ \frac{1}{2} & \frac{6}{5} & 0 \end{bmatrix}$$

iv) $B_1 = \{(1, 3, 4), (2, -5, 2), (-4, 2, -6)\}$, $B_2 = \{(1, 2, -2), (4, 1, -4), (-2, 5, 8)\}$

Solución:

$$\begin{aligned} [B_1/B_2] &= \left[\begin{array}{ccc|ccc} 1 & 2 & -4 & 1 & 4 & -2 \\ 3 & -5 & 2 & 2 & 1 & 5 \\ 4 & 2 & -6 & -2 & -4 & 8 \end{array} \right] \begin{array}{l} [-3f_1 + f_2 \rightarrow f_2] \\ [-4f_1 + f_3 \rightarrow f_3] \end{array} \sim \\ & \left[\begin{array}{ccc|ccc} 1 & 2 & -4 & 1 & 4 & -2 \\ 0 & -11 & 14 & -1 & -11 & 11 \\ 0 & -6 & 10 & -6 & -20 & 16 \end{array} \right] [-\frac{1}{11}f_2 \rightarrow f_2] \sim \\ & \left[\begin{array}{ccc|ccc} 1 & 2 & -4 & 1 & 4 & -2 \\ 0 & 1 & -\frac{14}{11} & \frac{1}{11} & 1 & -1 \\ 0 & -6 & 10 & -6 & -20 & 16 \end{array} \right] \begin{array}{l} [-2f_2 + f_1 \rightarrow f_1] \\ [6f_2 + f_3 \rightarrow f_3] \end{array} \sim \\ & \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{16}{11} & \frac{9}{11} & 2 & 0 \\ 0 & 1 & -\frac{14}{11} & \frac{1}{11} & 1 & -1 \\ 0 & 0 & 1 & -\frac{10}{11} & -14 & \frac{1}{0} \end{array} \right] [\frac{11}{26}f_3 \rightarrow f_3] \sim \\ & \left[\begin{array}{ccc|ccc} 1 & 0 & -\frac{16}{11} & \frac{9}{11} & 2 & 0 \\ 0 & 1 & -\frac{14}{11} & \frac{1}{11} & 1 & -1 \\ 0 & 0 & 1 & -\frac{30}{13} & -\frac{77}{13} & \frac{55}{13} \end{array} \right] \begin{array}{l} [\frac{14}{11}f_3 + f_2 \rightarrow f_2] \\ [\frac{16}{11}f_3 + f_1 \rightarrow f_1] \end{array} \sim \\ & \left[\begin{array}{ccc|ccc} 1 & 0 & 1 & -\frac{33}{13} & -\frac{86}{13} & \frac{80}{13} \\ 0 & 1 & 0 & -\frac{37}{13} & -\frac{85}{13} & \frac{57}{13} \\ 0 & 0 & 1 & -\frac{30}{13} & -\frac{77}{13} & \frac{55}{13} \end{array} \right] \end{aligned}$$

Por lo tanto

$$P_{B_1 \rightarrow B_2} = \begin{bmatrix} -\frac{33}{13} & -\frac{86}{13} & \frac{80}{13} \\ -\frac{37}{13} & -\frac{85}{13} & \frac{57}{13} \\ -\frac{30}{13} & -\frac{77}{13} & \frac{55}{13} \end{bmatrix}$$